HELSINKI SCHOOL OF ECONOMICS (HSE)
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IMPULSE CONTROL MODEL OF CORPORATE CASH MANAGEMENT

Jump-Diffusion Approach

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Sami Laine
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Tarkastajat:
TKT, Pauli Murto ja
Prof. Pertti Haaparanta

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ABSTRACT
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Objective

The objective of this study is to find an optimal, shareholder value maximizing, liquidity management policy for a given industrial corporation. In process of solving the optimal policy, characteristics of the underlying stochastic cash flow process are examined and relevant parameters estimated. Determinants of corporate cash holdings are discussed in the light of recent academic literature. Additionally, different cash management approaches are presented. Obtained results are subjected to an extensive sensitivity analysis.

Methodology

Time series properties are analyzed by calculating key parameters from the data and comparing goodness of fit of different distribution assumptions for an underlying stochastic process. An objective function-minimizing search algorithm is used to solve the two-sided trigger-target cash management policy.

Data

The time series data is collected from the data warehouse of the case corporation. Daily realizations of accounts receivable less accounts payable from 1.1.2005 to 31.12.2006 is used. Only the cash flow of the core business operations is taken into account. Additional information is collected from the case company's annual report for the financial years 2005 and 2006.

Results

In the light of recent literature, the precautionary and transaction cost motives of corporate cash management are concluded to reflect long- and short-term liquidity management problems respectively. The empirical results indicate that double exponential jump diffusion is a good characterization of the underlying stochastic process. Superior fit is obtained when small random movements of the cash flow are characterized by Brownian motion and larger shifts by compound Poisson process. A case-specific optimal cash management policy is found. When the lower trigger level is fixed, in the optimum the upper trigger level is 2.36 and the target return point 1.43 times this preset lower trigger. The upper trigger level and the optimal target return point are found to be increasing with regard to cash flow volatility and discount factor beta.

Keywords

cash management, impulse control, jump-diffusion.

HELSINGIN KAUPPAKORKEAKOULU — TIIVISTELMÄ

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Sami Laine

HYPPYDIFFUUSIOON POHJAUTUVA IMPULSSIKONTROLLI -MALLI YRITYKSEN KASSANHALLINNASSA

Tutkimuksen tavoitteet

Tämän tutkimuksen tavoitteena on löytää optimaalinen, osakkeenomistajan omistuksen arvoa maksimoiva kassanhallintamalli tutkimuksen kohdeyritykselle. Optimaalista kassanhallinnan menettelytapaa ratkaistaessa määritellään kassavirtaa kuvaava satunnaisprosessi ja estimoidaan prosessin olennaiset parametrit. Yritysten kassavaroihin vaikuttavia tekijöitä käsitellään viimeaikaisen tieteellisen kirjallisuuden pohjalta. Lisäksi tutkimuksessa esitellään erilaisia kassanhallintamenetelmiä. Saaduille empiirisille tuloksille suoritetaan laaja herkkyysanalyysi.

Tutkimusmetodit

Tutkimusaineiston aikasarjaominaisuuksia analysoidaan laskemalla olennaiset parametrit ja vertaamalla erilaisten jakaumaoletusten sopivuutta allaolevan satunnaisprosessin selittäjänä. Tavoitefunktiota minimoivaa hakualgoritmiä käytetään ratkaistaessa reagointi- ja kohdetasot määrittelevää kassanhallintamallia.

Tutkimusaineisto

Tutkimusaineistona käytetään kohdeyrityksen ydinliiketoiminnan päivittäisiä myyntisaatavia vähennettynä ostoveloilla ajanjaksolla 1.1.2005-31.12.2006. Tiedot on kerätty yrityksen perustietokannasta. Kohdeyrityksen vuosikertomuksia 2005 ja 2006 on käytetty lisämateriaalina.

Tutkimuksen tulokset

Viimeaikaisen tieteellisen tutkimuksen perusteella voidaan päätellä, että varovaisuus- ja transaktiokustannusmotiivit kuvaavat pitkän- ja lyhyenaikavälin kassanhallintaongelmaa. Empiiristen tulosten pohjalta voidaan todeta, että kaksoisexponentiaalinen hyppydiffuusio -malli kuvaa hyvin kassavirran satunnaisvaihtelua. Parhaiten sopiva malli saavutetaan, kun kassavirran vähäistä vaihtelua kuvataan Brownin liikkeellä ja suuria siirtymiä yhdistetyllä Poisson prosessilla. Tutkimuksessa ratkaistaan kohdeyritykselle yksilöllinen, optimaalinen kassanhallintamenetelmä. Kun alempi reagointitaso on määritelty, on optimaalinen ylempi reagointitaso 2,36 kertaa ja optimaalinen kohdetaso 1,43 kertaa alempaa reagointitasoa suurempi. Ylempi reagointitaso ja optimaalinen kohdetaso ovat kassavirran volatiliteetin sekä diskonttotekijä betan suhteen kasvavia funktioita.

Avainsanat

kassanhallinta, impulssikontrolli, hyppydiffuusio.

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I Introduction

1 Background and motivation

"Poor cash management hits profits" headlined Financial Times August 29, 2005 when reporting on a survey that indicates that cash unnecessarily tied up in working capital is costing £4 billion in net profits for UK firms annually. Attention of the financial press to the large cash holdings of corporations reflects the current academic view of the existence of optimal cash holdings for a firm. Intuition behind the theory is fairly simple. Any given company needs some amount of cash in order to operate. However, there must be a point where additional cash becomes a burden. The reasoning is straight forward, if one makes the reasonable assumption that a company's main objective is to maximize the shareholder value. Up to some optimal level, the additional cash is beneficial for operating the company and adds to the shareholder value. Above the optimal level, the shareholder would be better off by having the excess cash invested elsewhere.

This reasoning raises question not only about the optimum but also the sheer existence of a shareholder-value maximizing liquidity level and the possibility to manage corporate funds accordingly. Evidence, such as extensive cash causing poor investment decisions by Harford (1999), has turned academic interest towards corporate cash positions. Literature on liquidity management dates back to Keynes (1936) and the first optimization models to Baumol (1952) and Miller and Orr (1966). Although such groundbreaking works as Modigliani and Miller (1958) and Myers (1984) have discussed the question of corporate liquidity, it took until the end of the last century before the first comprehensive framework to study the phenomena saw daylight. Opler, Pinkowitz and Williamson (1999) defined the three most important theories to explain corporate cash holdings: *the trade-off theory*, *the pecking order theory* and *the free cash flow theory*. The trade-off theory states that an optimal level of cash holdings for a given firm exists, and it can be measured by comparing marginal costs and marginal benefits of holding liquid assets. The pecking order theory takes an opposite stand by arguing that cash holdings can be seen as a negative debt and therefore an optimal cash level does not exist. The free cash flow theory argues that the management of a firm has an incentive to build up cash to gain power over investment decisions.

The definition of profitability makes understanding the corporate liquidity position interesting for the shareholder. Removing excess cash from the working capital and using it for installment of outstanding debt obligations affects the profitability of the company and therefore increases shareholder value. This makes correct definition of optimal liquidity level appealing from the shareholder's point of view. A number of empirical studies have been recently conducted in order to explain motivations of corporate cash holdings according to the framework of Opler et al. (e.g. Pinkowitz and Williamson 2001; Dittmar, Mahrt-Smith and Servaes 2003; Ferreira and Vilela 2004). Findings are somehow mixed. Although the trade-off theory and the pecking order theory have opposite points of view towards optimal cash level, both theories gain support (Ferreira and Vilela 2004). Opler et al. (1999) find empirical evidence to target cash levels. However, they argue that companies hold amounts of liquidity that exceed the optimum, thus supporting the precautionary motive as the main determinant of corporate cash holding.

When the correct liquidity level has been defined, the optimal methods of cash management should be explored. The shareholder's possibility to participate in corporate management is restricted and usually not feasible. Therefore, preset conventions defining the method of managing liquid position should be in the interests of the operational management and the shareholders. Precautionary cash management models lean on dividends and share issues as tools of liquidity management (Anderson and Carverhill 2005). The inflexibility of these means opens the door for trigger-target models when optimizing liquidity in the short run. The trigger-target models (also known as Ss-models) study how a firm should split its holdings between liquid assets and marketable securities (Miller and Orr 1966; Hindler and Waldmann 2001; Green 2001; Premachandra 2004; Bar-Ilan et al 2004). By definition, the Ss-models are flexible and thus feasible for day-to-day cash management operations. Harrison et al.

(1983) show that an optimal solution exists for the trigger-target optimization problem. To solve a trigger-target model, one should first define the parameters of the underlying stochastic process. This thesis builds on these two main objectives. In order to solve the optimal liquidity management model, the characteristics of the cash flow are analyzed first.

In this study, the optimal short-term cash management model for a given company is explored. The case company of this study is a large publicly listed industrial corporation that operates in global markets. The firm has access to financial securities markets and operates there actively. The cash flow from the firm's operations generates cash surplus or deficit that the firm manages with a variety of financial instruments, i.e. commercial papers, revolving debt facility, short-term deposits and bank line of credit. Additional sources and uses of funds are bilateral debt agreements and dividends to the shareholders, respectively. The business environment of the case company includes large as well as small business operations. Smaller cash flows are generated from selling end products to private users and buying raw materials in small quantities. Simultaneously, however, the corporation faces significantly larger individual cash flows that present major shifts in the liquidity position of the company. The large cash flows are result of bulk purchases of raw materials and sales of end products to other large industrial operators. Opportunistic behavior derived from favorable commodity prices is another source of large individual cash flows, both in and out.

The majority of Ss-models lean on the assumption of Brownian motion (BM) as the underlying stochastic process for the cash flow. However, jump-diffusion presents superior results when defining the stochastic process that includes unexpected jumps up and down. The applicability of jump-diffusion models when characterizing stock-market and currency-market movements has led to a large amount of attention being devoted to the models. Pioneering work of Merton (1976) has set the foundation for these studies. Rosenfeld (1980) presents first attempts to estimate parameters of the jump-diffusion using *Maximum Likelihood Estimation* (MLE). MLE estimation can be a burdensome nonlinear optimization problem. In order to reduce computational demand, Hanson, Wesman and Zhu (2004) describe a *Multinomial Maximum Likelihood Estimation* (MMLE) method that builds on the second order estimation of the bin probability distribution.

This work has the following steps. First, the formal research objective of this study, together with some constraining definitions, is stated. In the second part, a general understanding of theories behind corporate cash holdings is established. After this, the theoretical foundations of corporate liquidity management are presented. The third section describes the mathematical methods of the parameter estimation of a jump-diffusion and the structure of the impulse control models. The data specific to the case company and its time series properties are introduced in section IV followed by the numerical solution and sensitivity analysis of the impulse control model in section V. Section VI summarizes the results and concludes the work. In each relevant part, the advantages and pitfalls of the chosen approaches are discussed individually.

2 Objective

The prime objective of this thesis can be stated in the following research question:

- What is the optimal cash management policy that maximizes the shareholder value for a given industrial corporation?

The prime objective can be further divided into three individual sub-objectives that construct the framework of this study. The sub-objectives are, (i) to choose a liquidity management model that is feasible within predetermined constraints and solvable in an empirical setting, (ii) to estimate or by other means define the parameters for the chosen liquidity model and (iii) to find a solution of the chosen model and study the sensitivity of the model with regard to the relevant parameters.

Some definitions are set in order to achieve the objectives listed above. First, the relevant time frame is set to a short-run analysis. Second, a plausible assumption of the shareholder-value maximization is set as the objective of the firm's liquidity management. Further on, the possibility of hedging the cash flows is considered irrelevant alongside questions related to foreign currency exchange and interest rate hedging. In addition, only two asset classes are considered to exist. Finally, this thesis is constrained to analyze the given cash flow optimization problem of a single case company. Therefore, no attempt to develop a generalized model has been made. This study is conducted as normative empirical research. The estimation method for the parameters of jump-diffusion by Hanson et al. (2004) and the Generalized Impulse Control Model of Cash Management by Bar-Ilan, Perry and Stadje (2004) are used as given tools. Therefore, no attempt is made to prove the derivation of these models formally. However, a comprehensive literature review of recent cash management research is presented.

Despite the company-specific approach of the study, this thesis also contributes to the prior academic research of optimal cash management policies on three accounts. First, the *transaction cost motive* approach (Baumol 1952; Miller and Orr 1966; Harrison et al. 1983; Hindler and Waldman 2001; Green 2001; Premachandra 2004; Bar-Ilan et al. 2004) and *precautionary motive* (Opler et al. 1999; Ferreira and Vilela 2004; Anderson and Carverhill 2005) are argued to represent short- and long-term solutions to a single liquidity optimization problem. Frenkel and Jovanovic (1980) have studied these two motives in the same framework but the long-run and short-run point of view has not been formerly introduced. Second, as far as the author is aware, there has not been a prior attempt to consolidate the study of an underlying stochastic process of a cash flow and optimal cash management policy to a single normative cash management policy. It is argued that such a comprehensive approach is achievable; nevertheless, results are highly case-specific. Finally, the comprehensive conclusions of Fenkel and Jovanovic (1981) and Bar-Ilan et al. (2004) are further extended by an argument that parameter choices of the optimal cash management policy reflect on corporations' share prices.

II Literature review

The prime objective of this work is to describe and solve a short-run financial planning problem of a specific firm. However, it is worthwhile to understand the theories underlying corporate cash holdings from more than one perspective. This work attempts to enlighten the issue by first introducing recent literature on the theoretical foundations of corporate cash management. After the general understanding of the theoretical ground is established, the firm-specific point of view can be tackled. A comprehensive study of corporate cash holdings by Opler et al. (1999) is used as a framework for the investor's view of the liquidity in the remainder of this chapter. The foundation for the rest of this study is established in this section by arguing that despite the recent findings supporting precautionary motive of liquid holdings (e.g. Opler et al. 1999; Ferreira and Vilela 2004), there is still room for transaction cost models, such as the trigger-target model, when analysing optimal cash management in the short run. The section concludes with an argument that the split between the transaction cost motive approach and the precautionary motive approach of corporate cash management could actually be seen as long- and short-term approaches to the same problem.

1 Determinants of corporate cash holdings

In the world of perfect capital markets described by Modigliani and Miller (1958), the market value of a firm is independent of the capital structure. Thus, the holding of liquid assets becomes irrelevant. The absence of the liquidity premium[1] results in no opportunity cost for cash holding. Therefore, a firm can borrow money to invest in liquid assets while the shareholder value remains unchanged. Under these conditions, there should be no reason for companies to have any cash holdings. However, in reality firms hold significant amounts of cash. Ferreira and Vilela (2004) report that listed

[1] In a world of significant transaction costs, assets that can be easily exchanged to cash are expected to have a lower return to reflect this benefit (Amihud and Mendelson 1986). From this follows that there is a cost of holding cash, i.e. liquidity premium.

EMU[2] corporations held as much as 15 percent of their total book value in cash or cash equivalents at the end of 2000. Dittmar et al. (2003) found that the worldwide cash holdings of the largest companies were 9 percent of the total book value. The scale of investments in cash indicates the importance of liquid assets for corporations. Large cash holdings of corporations have also been noticed outside of the academic literature. For example, The Financial Times and leading Finnish financial newspaper Kauppalehti have published a number of articles about the extensive cash holdings of corporations[3].

There is comprehensive literature available on the theories on cash holdings. Foundations of the cash management research were established by Keynes (1936) who introduced the transaction cost and precautionary motives as the main drivers for holding liquid assets. Intuition is fairly simple. A company can hold cash in order to minimise the costs of monetary transactions (transaction cost motive) or hold liquidity in order to cope with the uncertainty of future business operations (precautionary motive). First attempts in the search of cost-minimizing optimization models date back to William J. Baumol's (1952) inventory model and the Miller-Orr model by Merton Miller and Daniel Orr (1966). Dittmar et al. (2003) argue that until recently only transaction costs were assumed to be major determinants for corporate cash holdings. Opler et al. (1999) considerably expanded the evidence by defining a theoretical framework that can be used to study the motives of holding money by firms. The following chapters present in detail the three most relevant theories to explain which corporate characteristics influence the cash holdings decisions; i.e. the trade-off theory, the financial hierarchy theory - also known as the pecking order theory, originally introduced by Myers (1984) - and the free cash flow theory that follows the theoretical foundation of Jensen (1986). The discussion concludes with remarks on findings of some of the most recent empirical literature and the lack of consensual matter among the theories.

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[2] The EMU includes the following countries: Germany, France, Netherlands, Italy, Spain, Finland, Belgium, Austria, Ireland, Luxemburg, Greece and Portugal.

[3] "*Poor cash management hits profits*" Financial Times August 29, 2005.
"*Yritysten kassat hipovat ennätystä*" Kauppalehti August 18, 2005.
"*Nokia jatkaa kassan siirtämistä omistajille*" Kauppalehti January 27, 2006.

1.1 *The trade-off theory*

The basic assumption underlying the trade-off theory is that the management of a company evaluates the marginal benefits (MR) and marginal costs (MC) of cash holdings. The shareholder-value maximising managers set MR = MC. Using the framework of Keynes (1936), Oppler et al. (1999) describe two main benefits of cash holdings. First, by holding liquid assets, the firm saves transaction costs as it does not have to raise funds or liquidate assets to meet its obligations. Thus, cash can be seen as a buffer between the corporate sources and uses of funds. Second, liquid holdings give the management ready access to additional investments without entering external financing markets that can be costly due to information asymmetries and agency cost of debt. Without disposable cash, the management may be forced to pass positive NPV projects because they are reluctant to raise external financing. Therefore, by holding liquid assets, the management saves costs arising from foregone investment opportunities. In addition, Ferreira and Vilela (2004) argue that cash holdings acting as a safety reserve reduce the likelihood of financial distress and allow optimal investment policy, even when financial constraints such as leverage are met.

The main cost of holding cash comes from the low rate of return caused by the liquidity premium. Liquid holdings have an opportunity costs; i.e. the rate of return for cash and cash equivalents do not meet that of other investments with the same risk. This is often called cost-of-carry (Dittmar et al. 2003, 115). *Table 1* concludes the view of the relevant corporate characteristics that, according to the trade-off theory, influence the cash holdings decisions and the expected sign of the effect.

Table 1: Corporate characteristics and expected sign according to trade-off theory

Corporate characteristic	*Relation on cash holdings*
Dividend payments	Firm that pay dividends can raise cash with low cost by reducing dividends. Thus, dividend-paying firms hold less cash.
Investment opportunity set	Better investment opportunities result in a higher level of cash due to the higher expected loss resulting from giving up an investment.
Liquid asset substitutes	Extensive liquid asset substitutes result in a lower level of cash.
Leverage	Firms with high leverage are expected to hold more cash in order to reduce the probability of financial distress. On the other hand, leverage ratio indicates a firm's ability to raise debt; thus, the firm is able to hold less cash. Thus, the relationship is ambiguous.
Size	Economies of scale in cash management suggested by Miller and Orr (1966) result in a relatively lower level of cash for bigger companies.
Cash flow	Cash flow as a source of liquidity results in a lower level of cash
Cash flow uncertainty	An increasing probability of cash shortage results in a higher level of cash
Debt maturity	Relation not clear

(Ferreira and Vilela 2004, 298-299.)

According to the trade-off theory, there is an optimal cash level for a firm. Thus, it is possible to state whether the firm is maximising the shareholder value by analysing the costs and benefits of the corporate cash holdings. However, Opler et al. (1999) argue that managers and shareholders can view optimal cash holdings differently. Managers often place too much weight on a precautionary motive for holding cash. Imperfect cash holdings could also be explained by the Agency theory. Mayers and Majluf (1984) argue that asymmetric information between management and investors results in the management preferring internal finance over information-sensitive external finance.

1.2 *The pecking order theory*

Opler et al. (1999) also introduce an alternative view to the trade-off model. In the pecking order theory, the size of cash holdings becomes irrelevant. They argue that nothing changes in a corporation if it holds an additional (small) amount of cash financed by an equal amount of debt. Even with optimal capital structure and resulting optimal amount of net debt, there cannot be an optimal level of cash holding because cash can be seen as negative debt. The same reasoning holds with the pecking order theory of Myers (1984), supported by the theoretical foundations of Myers and Majluf (1984). The pecking order model (also know as the financing hierarchy model) states that in order to minimise adverse selection costs, the firm should finance investments first with retained earnings, then with debt and finally with external equity. Analogously, the firm uses accumulated cash first to pay back a debt. Therefore, a firm that is not constrained in its investment policy, uses cash flow to increase its cash holdings unless it has a debt to pay. Opler et al. (1999) argue that according to financing hierarchy model, changes in cash holdings are results of changes in a firm's internal resources. Dittmar et al. (2003) support this view and state that cash balances are just an outcome of a firm's investment and financial decisions. According to Ferreira and Vilela (2004), the pecking order theory suggests that firms use cash as a buffer between retained earnings and investment needs. *Table 2* lists relevant corporate characteristics, according to the pecking order theory, influencing cash holdings decisions and the expected sign of the effect.

Table 2: Corporate characteristics and expected sign according to the pecking order theory

Corporate characteristic	*Relation on cash holdings*
Investment opportunity set	Large investment opportunity set results in large cash holdings in order to avoid costly external financing.
Leverage	Debt grows and cash holdings fall when investment exceeds retained earnings thus, relation is expected to be negative.
Size	Larger firms have presumably been successful and hence should have more cash.
Cash flow	It is expected that companies with a high cash flow hold more cash.

(Ferreira and Vilela 2004, 300.)

The main difference between the trade-off theory and the pecking order theory derives from the relation between cash holdings and investment decisions. The former predicts the relation to be positive while the latter theory predicts it to be negative. Furthermore, the pecking order theory sees cash as a negative debt, which leads to the absence of an optimal cash level.

1.3 *The free cash flow theory*

When extending the financial hierarchy model to explain corporate cash holdings, one faces rather restrictive conditions in order to be consistent with the shareholder wealth maximisation (Opler 1999). Intuitively, for a firm accumulating an extensive amount of cash, there should be some point of liquidity where the shareholders would be better off receiving additional dividends. The existence of extensive cash reserves can be explained by the free cash flow theory of Jensen (1986). The theory suggests that managers have an incentive to build up cash holdings in order to gain discretionary power over the corporate investment decisions. Ferreira and Vilela (2004) argue that having available cash holdings removes the management's obligations to provide the capital markets detailed information on investment projects. Opler et al. (1999) state that if the management is reluctant to pay excess cash to the shareholders due to reasons discussed by Jensen (1986), there is empirical evidence supporting the financing hierarchy view even though there is no optimal shareholder-value maximising amount of cash. *Table 3* lists the relevant corporate characteristics, according to the free cash flow theory, influencing cash holdings decisions and the expected sign of the effect.

Table 3: Corporate characteristics and the expected sign according to the free cash flow theory

Corporate characteristic	*Relation on cash holdings*
Investment opportunity set	Managers of firms with poor investment opportunities are expected to hold more cash to ensure availability of funds to invest in growth projects.
Leverage	High-leverage firms face more monitoring allowing less managerial power. Thus, high leverage results in less cash holdings.
Size	Managers of larger firms are expected to have more discretional power due to shareholder dispersion. Thus, cash holdings are positively related to size.

(Ferreira and Vilela 2004, 300.)

It is not surprising that cash holdings have gained a great degree of attention in the recent empirical literature. It is in the interest of the shareholders to understand whether firms are holding adequate levels of cash. Harford (1999) finds evidence that firms with "excess cash" use it in poor acquisitions even in the U.S. where shareholders are well protected. On the other hand, Opler et al. (1999) state that there is little evidence of money "burning a hole in managements' pockets". Opler et al. (1999) find empirical evidence supportive to target levels of cash but argue that firms that do well accumulate more cash than would be expected by the static trade-off model. They argue that the motivation behind the behaviour derives from an excessively strong precautionary motive. Anderson and Carverhill (2005) explain the behaviour with a "dynamic trade-off" model. In the dynamic model, the optimal cash level fluctuates according to the expected future cash flows.

Corporate cash holdings and the influence of corporate characteristics have been tested with a variety of data sources. Kim et al. (1998); Opler et al. (1999); Pinkowitz and Williamson (2001); Dittmar et al. (2003); Ferreira and Vilela (2004), among others, have studied whether the relevant corporate characteristics influencing and the prediction of the expected sign gain empirical support for some or all of the theories described above. *Table 4* summarises the relevant corporate characteristics and predicted signs for each theory. As one can easily see, there is no consensus between the theories. For example, when the investment opportunity set is predicted to have a positive effect on cash holdings according to the Trade-off theory and the Pecking order theory, the expected effect is negative according to the Free cash flow theory.

Table 4: Summary of predictions

		Expected sign	
Variable	*Trade-off theory*	*Pecking order theory*	*Free cash flow theory*
Dividend payments	-		
Investment opportunity	+	+	-
Liquid asset substitutes	-		
Leverage	?	-	-
Real size	-	+	+
Cash flow uncertainty	+		
Cash flow	-	+	
Debt maturity	?		

(Ferreira and Vilela 2004, 301.)

The results of extensive empirical research are also mixed. Opler et al. (1999) suggest, in the study of non-financial publicly traded U.S. firms reported in *Compustat* from 1971 to 1994, that firms with strong growth opportunities and higher business risk as well as small firms hold more cash. They also argue that firms with access to the capital markets, such as large corporations and those with credit ratings, as well as high-levered firms hold less cash. Opler et al. (1999) also find evidence that the management accumulates excess cash if it has the opportunity to do so. When comparing Japanese and U.S. firms, Pinkowitz and Williamson (2001) find evidence that powerful banks result in increasing cash holdings among companies. Dittmar et al. (2003) analysed in 1998 the data of 11,000 international companies from *Global Vantage database* and found strong support for the corporate governance issues determining cash levels. Firms operating in countries with the lowest level of shareholder protection hold as much as 25 percent more cash than firms in countries with a high level of shareholder protection. Ferreira and Vilela (2004) studied publicly traded firms from the EMU countries from

1987 to 2000 obtained from *Datastream* and added two distinct dimensions to the existing evidence. First, contrary to Pinkowitz and Williamson (2001), they argue that there is a significant negative relationship between bank debt and cash holdings. Second, their observations support the Dittmar et al. (2003) findings of larger cash holdings of firms operating in countries with inferior investor protection mechanisms. However, they do not confirm the Dittmar et al. (2003) evidence of positive impact of capital market development to cash holdings. In fact, their findings are the opposite. To reinforce previous findings, Ozakan and Ozakan (2004) provide evidence, from a sample of 1,029 publicly traded UK firms from *Datastream* from 1984 to 1999, that cash flows and growth opportunities have a positive impact, and liquid asset substitutes, leverage and bank debt have a negative impact on cash holdings. They also suggest that the ownership structure plays an important role in determining the cash holdings, especially the non-monotonic relationship of managerial ownership and corporate cash holdings and the irrelevance of the board composition.

As *Table 4* indicates and empirical results confirm, there is no consensus among the theories introduced in this chapter. E.g. the findings of Oppler et al. (1999), Ozakan and Ozakan (2002) and Ferreira and Vilela (2004) are consistent with the trade-off theory and the pecking order theory. However, the results contradict the free cash flow theory. Oppler et al. (1999) provide support to the static trade-off model but also point out that it is sometimes difficult to distinguish between financial hierarchy and trade-off models. Ferreira and Vilela (2004) conclude that it is both the trade-off and pecking order theories that play an important role in explaining the determinants of corporate cash holdings.

2 Corporate liquidity management

The diverse discussion above gives a broad view of the determinants of corporate cash holdings, binds theoretical framework to the issue and gains some support with empirical findings. Evidence that firms in fact do have target cash levels (Opler et al.

1999; Ozakan and Ozakan 2002; Ferreira and Vilela 2004) makes it appealing to look for the optimal liquidity management model from the firm's point of view.

There is substantial evidence that the observed cash levels are above the prediction of the static trade-off models, supporting an argument that the precautionary motive of cash holding dominates the more traditional view of the transaction cost motive (Opler et al. 1999.) Nevertheless, it is argued here that there is still room for models that are based on the transaction cost motive. As the time horizon for cash management decisions is short, the precautionary motive models are not feasible. Therefore, cash managers should lean on the impulse control models when looking for an optimal cash management policy. The remainder of this chapter discusses corporate cash management in general terms, describes the framework for analysing the optimal cash level according to the trade-off model and introduces optimal cash holding models based on the precautionary motive and the transaction cost motive. The chapter concludes with a discussion of the relevance of the theories and an argument supporting usability of each model within the relevant time frame.

2.1 *Corporate cash management*

Corporate finance text books dedicate significant space for describing financial planning and cash management issues. Brealey and Myers (2003) argue that successful cash management is based on the efficient means of cash collection and disbursement that generatse inflows and outflows of cash which evolve into a cash surplus or deficit that should be managed. Arguments behind the importance of effective cash management become straightforward with a plausible assumption of shareholder value maximisation.

The profitability of a firm can be measured by dividing the profits earned by assets used in the process. By multiplying and dividing the turnover, one arrives at the following definition of profitability:

$$\begin{aligned} \textit{profitability} &= \textit{profit/turnover} * \textit{turnover/assets} \\ &= \textit{profit margin} * \textit{capital turnover rate} \end{aligned}$$

Dolfe and Koritz (1999) state that good cash management will improve both determinants of the profitability thus having an immediate effect on the shareholder value. Reducing liquid holdings improves the capital turnover rate. Furthermore, the released capital can be used for investments or for repaying debts. This will lead to interest rate savings thus improving the profit.

The cornerstone of efficient liquidity management is liquidity forecasting and planning. Short-term forecasts are performed in order to ensure that sufficient liquidity reserves are always available while long-term forecasts provide the management with information for determining the required liquidity reserves and for optimising the capital structure and the investment decisions (Dolfe and Koritz 1999). According to Cooper (2004), a short-term forecast covers approximately 30 days and a long-term forecast from three to five years. Additionally, he introduces medium-term forecasts that lie between the short- and long-term ones. Kallberg, White and Ziemba (1982) argue that all business firms experience short-term cash management problems. Cooper (2004) follows suit by stating that most companies accept the need for accurate cash flow forecasts, but few are able to deliver them. He adds that while companies can forecast total inflows and outflows of money for the year as whole, it is extremely difficult to achieve accurate cash flow forecast on a short-term basis. Therefore, the working capital in some companies can fluctuate substantially (Cooper 2004, 327), if one neglects the possibility to forecast in the cash management model and concentrates solely on the random fluctuation of the cash flow. The results obtained will represent the extreme values of efficient cash management. The obtained cash policy could be further "tightened" by using the difference between realized and forecast cash flow as the source of the uncertainty in the cash flow.

2.2 *General framework of cash holdings*

According to the trade-off theory as a determinant for corporate cash holdings, optimal holdings of liquid assets can be analysed in a framework of marginal costs and marginal benefits. Holding an additional liquid asset reduces the probability of a shortage of liquid assets, therefore decreasing the cost of being short of cash under the reasonable assumption that liquid assets have decreasing marginal benefits (Opler et al. 1999; Ferreira and Vilela 2004). The cost of liquid assets derives from the lower expected return. This cost does not vary with the amount of assets held.

Figure 1 describes the optimal liquid asset holding decision. For a given amount of liquid assets,

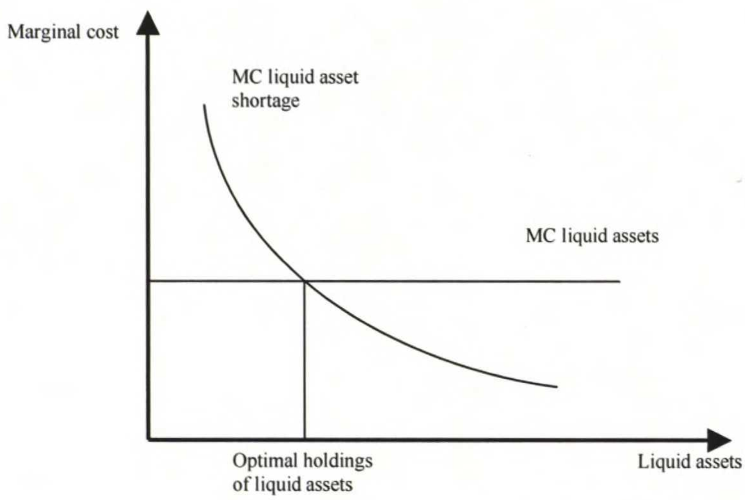


Figure 1: Optimal holdings of liquid assets (Opler et al. 1999, 8).

an increase in the cost or an increase in the probability of being short of liquid assets shifts the marginal cost curve of liquid asset shortage to the right, increasing the optimal level of liquid asset holdings. The increasing probability of being short can derive e.g. from increasing volatility of cash flows or increasing uncertainty of future incomes.

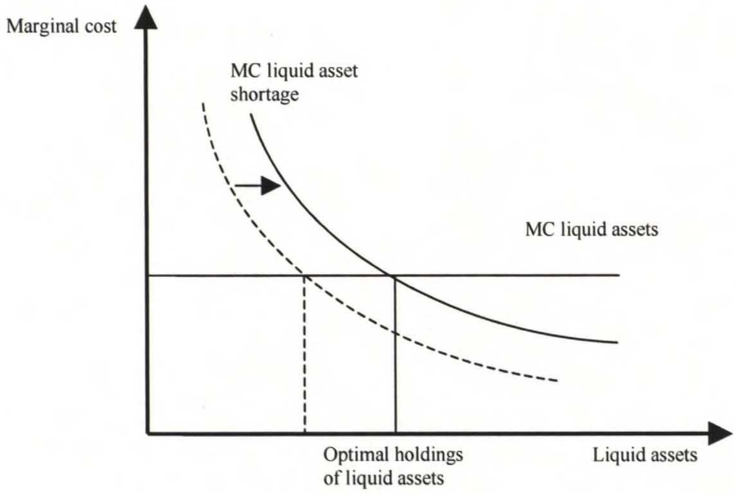


Figure 2: Adjusted optimal holding of liquid assets

2.3 *The transaction cost motive*

The transaction cost motive has dominated the research of corporate cash management (Han and Qui, 2006, 1). The original trigger-target models of Baumol (1952) and Miller and Orr (1966) have been widely studied in the cash flow management literature and a number of enhancements have been made to the models. Recently, Hindler and Waldmann (2001) introduced the trigger-target model with a randomly varying environment. The possibility of varying environmental factors, such as the interest rate level or commodity prices, is integrated into the model. Green (2001) studies the dynamics of aggregate money holdings by allowing the trigger and target levels to fluctuate. Premachandra (2004) argues that the diffusion approximation model is a superior alternative to the Miller-Orr model by returning smaller errors in managing cash. Bar-Ilan et al. (2004) present a generalized impulse control model of cash management where the traditional view of a single stochastic component underlying the cash flow is discarded and a more general view of the jump-diffusion process is introduced.

The trigger-target models study the split between a firm's holding of marketable securities and liquid assets based on the firm's needs for cash. The model returns an optimizing problem of cash holdings with regard to the predictability of needs for cash, interest rate of marketable securities and transfer costs. Harrison, Shalke and Taylor (1983) show that an optimal solution for such an optimization problem exists if the underlying stochastic process is described as a Brownian Motion. Bar-Ilan et al. (2004) add that an optimum can be also found if the jump-diffusion describes the stochastic process.

The trigger-target models can be divided into two broad categories. First, the models dealing with household money demand were originally introduced by Baumol (1952). In the model, the money stock is a downward drifting flow of expenditures. When a certain cash level is reached, it triggers a conversion of financial assets to raise the money stock to the target level. Baumol (1952) defines the optimal amount of withdrawals C as a function of steady stream of payments T, interest i, and a fixed "broker's fee" of b. The optimal solution of C is also a familiar result of the optimal order quantity from operations management text books (see e.g. Krajewski, Rizman & Malhotra 2006).

(1)
$$C = \sqrt{\frac{2bT}{i}}.$$

The second category of models considers the cash management of firms. The main difference is the fact that firms have daily inflows of money as well as daily outflows. The model pioneered by Miller and Orr (1966) establishes a framework of upper and lower trigger levels of cash holdings. When the trigger level is reached, the firm either uses extra funds to buy marketable securities or obtains new funds by selling securities to return to the target level(s) of cash holdings. The original Miller-Orr model is based on some simplified assumptions i.e. a constant daily interest rate ι, constant transaction costs δ, a negligible lead-time in portfolio transfers, and net cash flows that can be characterized as independent Bernoulli trials with equal probability. By minimising the expected costs, they obtain the following optimal values:

(i) Optimal return point: $\chi^* = \alpha + \phi^*$
(ii) Optimal upper limit: $\beta^* = \alpha + 3\phi^*$

where $\phi^* = (3\delta\sigma^2/4\iota)^{1/3}$, α is a constant. σ^2 represents the variance of the daily cash balance.

Premachandra (2004) argues that when relaxing some of the unrealistic assumptions of the Miller-Orr model and using the diffusion[4] approximation model, they derive superior results compared to Miller and Orr. They state that numerical simulations result in an error percentage (% = 0.61 – 6.63) of managing the cash with regard to Miller-Orr model (% = 0 – 43.39). Hindler and Waldman (2001) and Bar-Ilan et al. (2004) go even further in the quest of generalizing the model.

Hindler and Waldmann (2001) state that previous optimal conditions for the transfer rule strongly depend on the assumption of independent and identically distributed (i.i.d.) cash flows. They argue that in practice, dependencies and sources of uncertainty are observed. They also argue that business environments affect cash holding decisions. Cash flows respond to economy-wide variables, such as interest rates, and industry wide variables, such as commodity prices. Hindler and Wadmann (2001) model trigger-target rules with additional dependencies and sources of uncertainty where the transfer rule $f(s_n, i_n)$ does not depend only on the cash balance s_n but also on the environment i_n. The discount factor β is also set to depend on the environmental states. Hindler and Waldmann (2001) derive the optimal decision rule f^* for suitable constraints and state structural results (based on convex holding and transfer costs) with regard to the environment i:

[4] Diffusion is a physics term for the phenomenon of movement of particles from an area where their concentration is high to an area that always has a low concentration. Brownian motion is a specific type of diffusion named in honor of the botanist *Robert Brown*.

(i) $f^*(s,.), s \in S$, is increasing in i

(ii) in case of proportional transfer costs $S^{\pm}$ is increasing in i.

Bar-Ilan et al. (2004) contribute to the literature on cash management by discarding the existing view of describing cash flows as a Brownian motion (BM) or a Compound Poisson Process (CPP). They use superposition of BM with continuous infinitesimal movements and CPP with positive and negative jumps to explain the cash flow. Due to its applicability to stock exchange and foreign currency movements, the jump-diffusion process has been widely studied (see e.g. Merton 1976; Ramezani and Zeng 1999; Hanso et al. 2004). Stochastic processes are discussed in detain in chapter *III 1*. The following chapter presents recent studies that model optimal cash management according to the precautionary motive.

2.4 *The precautionary motive*

As the discussion above points out, the recent literature on corporate cash management has mainly dealt with the transaction cost motive as the main determinant of corporate cash holdings. However, Opler et al. (1999) argue that it is the precautionary motive that seems to dominate cash holding decisions in corporations. Firms hold higher liquidity levels than the static trade-off model predicts, and the main usage of excessive cash holdings is argued to be for operating losses rather than capital acquisitions. Ferreira and Vilela (2004) support this view. However, Han and Qui (2006) point out that although the precautionary motive has gained support, it has not been adequately modelled in the literature. Anderson and Carverhill (2005) also argue that the finance theory gives very little quantitative guidance to the question of adequate level of cash holdings.

When studying precautionary behaviour of corporations, different types of hedging strategies are usually counted for. In their study of optimal hedging strategies, Mello and Parson (2000) provide theoretical treatment of future cash flows and the variance related to the cash flow. Roched and Villeneuve (2004) analyze the hedging and insurance of a corporation facing a liquidity risk. Both papers show that hedging improves corporate value by reducing optimal liquidity holdings. Acharya et al. (2002) study strategic debt-service and present a model of precautionary liquidity holding by a levered company. Han and Qui (2006) build a link between a firm's cash holdings, cash flow uncertainty and financial constraints. They argue that a financially constrained firm increases its cash holdings in response to an increase in its cash flow volatility. In contrast, cash flow volatility is irrelevant for financially unconstrained firms. Anderson and Carverhill (2005) study a continuous time model of the cash holding of a levered company with cash flows fluctuating with business conditions. They argue that the "dynamic trade-off" model presented in their working paper is able to explain the findings of Opler et al. (1999).

Anderson and Carverhill (2005) consider a levered firm financed by equity and long-term debt with assets in place generating a random cash flow. They take fluctuations of business conditions into account by supposing that the rate of revenue flows is a stochastic process reverting toward a long-term mean. A firm can use its net profits to pay out dividends or to retain accumulated funds as liquid assets. The possibility of bankruptcy is taken into account in the model. The firm can obtain additional funds by issuing securities; liquid assets are a precaution against poor business conditions, carrying liquid assets is costly, and agency costs make issuing new shares inefficient. In this setting, the decision problem - assumed to be under the control of equity value-maximising shareholders - is the amount of dividends to be paid and the number of new shares to be issued. The decision will determine the liquid asset holding for the firm. Anderson and Carverhill (2005) also analyse the model by implementing a numerical benchmark case and analysing the model's sensitivity to the most significant economic parameters. They conclude that the optimal policy for the firm is to target a level of liquid assets that varies according to the level of expected cash flows. The optimal policy is not monotonic. They also find that the basic parameters affecting debt and equity value are also the main drivers of optimal cash policy. To support the findings of Oppler et al. (1999), Anderson and Carverhill (2005) argue that it is in the shareholders' interest to hold relatively large amounts of cash inside the firm if the firm has access to efficient capital markets. The comprehensive model generates realistic numbers for leverage, average cash holding, equity volatility, yield spreads, probability of default, and loss given default.

As the profound discussion in the recent cash management literature above demonstrates, studies of corporate cash holding seem to be divided along with the original division by Keynes (1936). The argument of this work is that in the light of recent literature, the initial motives could be seen not as two separate ones but rather a division between short-term and long-term motives of corporate cash holdings. The following discussion will enlighten the argument.

The cash management operations described by Anderson and Carverhill (2005) could be seen as strategic decisions that are not feasible on a frequent basis. The intuition is self-explanatory: due to the high menu costs, dividends and equity issuing cannot be used for operative cash management. There is also some empirical evidence that firms are reluctant to change their dividend policies due to the informational content of the dividend changes (Bernartzi, Michaely and Thaler 1997). Therefore, the model should not be used as a guideline for short-term liquidity management. However, the model yields a significant amount of information concerning medium or long-term liquidity decisions and target levels of cash holdings. The findings of Opler et al. (1999) also support the reasoning. The data used by Opler et al. (1999) (also Ferreira and Vilela 2004) is gathered from interim reports and financial statements that should reflect strategic, rather than operative, decisions of the firm. Thus, the model of Anderson and Carverhill (2005) supported by earlier empirical findings should be interpreted as a long-term strategic liquidity management model for a corporation.

On the other hand, there is still room for trigger-target models in the corporate liquidity management. The Ss-models described above strongly lean on the possibility of managing cash reserves on a daily basis. However, daily cash management is, by all means, out of the shareholders' scope. It would not be economically efficient for shareholders to make operative decisions for the firm. Thus, the trigger-target models should be seen as short-term liquidity management models. A restriction of the models to consider only cash and marketable securities and deliberately leave out the possibility of paying dividends and issuing new equity outside the model, supports the view of the short-term model.

The corporate liquidity problem can therefore be argued to be a two-stage model. Dynamic models, such as the one introduced by Anderson and Carverhill (2005), determine the long-term target level of corporate liquidity, and the trigger-target models, such as the impulse control model of Bar-Ilan et al. (2004), define the range for short-term cash management policy around the long-term target.

III **Methodology**

This section introduces the theoretical background of the mathematical methods used in this thesis. The first two chapters focus on stochastic processes and methods of estimating the parameters of these processes. First, the stochastic processes with different characteristics are defined. This is followed by a definition of the log-likelihood function for a jump-diffusion process. Furthermore, the general properties of the Maximum Likelihood Estimation (MLE) and the Multinomial Maximum Likelihood Estimation (MMLE) by Hanson et al. (2004) are presented. Finally, the practical steps to carry out a second-order estimation for bin probability distribution for jump-diffusion are introduced. The final chapter discusses the basic principles of impulse control models and introduces in detail the *Generalized Impulse Control Model of Cash Management* by Avner Bar-Ilan, David Perry and Wolfgang Stadje (2004).

1 Stochastic process

Let C_t be defined as the level of cash holdings at time t. The assumption that a change in the level of cash holding follows geometric Brownian motion (GBM):

$$(2) \qquad \frac{dC_t}{C_t} = \mu_d dt + \sigma_d dz_t,$$

where μ_d is a constant drift parameter per unit of time, σ_d is a constant volatility of the cash flow and dz is a standard *Wiener process*[5], implies that the logarithm of price changes is normally distributed[6] with a mean $\mu_{ld} = \mu_d - 0{,}5\sigma_d^2$ and variance σ_d^2:

[5] The Wiener process is a special type of the Markov process with a mean change of zero and a variance rate of 1.0. The Wiener process is sometimes referred to as Brownian motion. (Hull 2006, 265-269.) The Markov process is a stochastic process where the past history of the variable is irrelevant (Hull 2006, 263).

[6] Complete derivation of lognormal property is given in Appendix (A2)

$$\ln(C_t / C_{t-1}) \sim N\left[\left(\mu_d - \frac{\sigma_d^2}{2}\right), \sigma_d^2\right], \quad (3)$$

where subscripts *d* and *ld* stand for diffusion and log-diffusion respectively. GBM could be described as a stochastic process where the general direction of the development is known, but at any given time, the next realization is completely random. Brownian motion is an underlying assumption for many well-known financial applications such as the trigger-target cash management model of Miller and Orr (1966) and the groundbreaking Black and Scholes (1973) option-pricing model.

In contrast to the Brownian motion assumption, financial data often includes isolated shifts up and down resulting from unpredictable information released to the market. Brownian motion approach lacks the possibility to accommodate such jumps in a time series. Jorion (1988) argues that these discontinuities in the stochastic process could be modelled by a jump-diffusion,

$$\frac{dC_t}{C_t} = \mu_d dt + \sigma_d dz_t + dq_t, \quad (4)$$

where dq is an independent Poisson process that is characterized by a mean number of jumps per unit time λ and jump size V. The process follows a standard Brownian motion until a jump of given magnitude up (down) shifts the process to a new level. Jumps are realized at Poisson times with the parameter λ. The jump magnitude V is here at first assumed to be lognormally distributed $V \sim lognormal(\theta, \delta^2)$. This representation, originally introduced by Merton (1976), allows the cash level to jump up (and down) in a non-fixed magnitude. In discrete time, this can be written as:

$$\ln(C_t / C_{t-1}) = \mu_{ld} + \sigma_d z + \sum_{i=1}^{n_i} \ln V_i, \quad (5)$$

where z is a standard normal deviate and n_i is the actual number of jumps. The jump diffusion model with a lognormally characterised jump component is an improvement to a standard BM approach. However, Merton's (1976) model does not accommodate for asymmetries of financial data.

Ramezani and Zeng (1999) describe a jump-diffusion model where the jump component is divided into independent up- and down-jumps representing "good" and "bad" surprise information respectively. In order to model the limited liability boundaries of the stock price - i.e. price cannot go below zero - Ramezani and Zeng (1999) use Pareto and Beta distributions to describe the up and down movements respectively. A model with independent distributions up and down can be written as:

(6) $$\frac{dC_t}{C_t} = \mu_d dt + \sigma_d dz_t + \sum_{j=u,d} (V^j_{P^j(\lambda^j t)} - 1) dP^j(\lambda^j t),$$

where V^j is the jump magnitude and $P^j(\lambda^j)$ independent Poisson processes with intensity parameters λ^j. Superscripts u and d represent up- and down-jumps respectively. Discrete time representation of the process is:

(7) $$\ln(C_t / C_{t-1}) = \mu_{ld} + \sigma_d z + \sum_{i=1}^{P^u(\lambda^u)} \ln V_i^u + \sum_{i=1}^{P^d(\lambda^d)} \ln V_i^d,$$

where V^u and V^d represent independently and identically distributed jump magnitudes. The limited liability boundaries of Ramezani and Zeng (1999) are relaxed in the double exponential jump diffusion model, where jump magnitudes are characterised to follow exponential distributions.

According to Bar-Ilan et al. (2004), the cash flow process of a corporation follows the superposition of Brownian Motion (BM) and Compound Poisson Process (CPP) where jump magnitudes follow independent exponential distributions. I.e. V_i of (5) is a sequence of i.i.d. jumps such that $Y_i = log(V_i)$ has an asymmetric double exponential distribution for up and down jumps.

(8) $$\log(V) = Y = \begin{cases} \xi^{+}, & \text{with probability p} \\ -\xi^{-}, & \text{with probability q} \end{cases},$$

where $p,q \geq 0$, $p+q=1$ are probabilities of up and down jumps, and ξ^{+} and ξ^{-} are exponential random variables with mean $1/\eta_1$ and $1/\eta_2$ respectively. This double exponential jump-diffusion model (DEJD) was also proposed by Kou (2002) to explain an empirical phenomenon of volatility smile[7] in option markets.

Kou (2002) argues that with minor intervals, such as daily observations, the change of the cash level can be approximated in distribution, and the density function (9) below is an approximation for $\Delta C(t)/C(t)$:

(9)
$$g(x) = \frac{1-\lambda\Delta t}{\sigma_d\sqrt{\Delta t}}\varphi\left(\frac{x-\mu_d\Delta t}{\sigma_d\sqrt{\Delta t}}\right) + \lambda\Delta t\begin{Bmatrix} p\eta_1 e^{(\sigma_d^2\eta_1^2\Delta t)/2} e^{-(x-\mu_d\Delta t)\eta_1}\Phi\left(\dfrac{x-\mu_d\Delta t-\sigma_d^2\eta_1\Delta t}{\sigma_d\sqrt{\Delta t}}\right) \\ +q\eta_2 e^{(\sigma_d^2\eta_2^2\Delta t)/2} e^{(x-\mu_d\Delta t)\eta_2}\Phi\left(-\dfrac{x-\mu_d\Delta t-\sigma_d^2\eta_2\Delta t}{\sigma_d\sqrt{\Delta t}}\right) \end{Bmatrix}.$$

Here $\varphi(\cdot)$ is the standard normal density function and $\Phi(\cdot)$ the cumulative distribution function. Thus, the log-likelihood function, given M equally spaced changes in the cash flow is:

(10) $$L(D; p,q,\eta_1,\eta_2,\mu_d,\sigma_d) = \sum_{i=1}^{M}\ln(g(x_i))$$

where D = {C(0), C(1),...C(M)} denotes the realization of cash holdings at equally spaced times. When the likelihood function is known, the maximum likelihood estimation method can be used to find parameters for the underlying stochastic process.

[7] A plot of implied volatility of an option as a function of its strike price is know as a volatility smile. This phenomenon implies that traders consider that the lognormal property underlying the option pricing models understates the probability of extreme values. (Hull 2006, 375-378.)

2 Likelihood estimation

Assuming that the given data set describes the independent realisations of some known distribution, the parameters of the underlying distribution can be estimated by using the maximum likelihood method of estimation[8] (MLE). If $P(X_i\ ;\ \theta)$ is the probability function of a sample observation X_i, where θ is the unknown parameter. Assuming independence of the sample observations X_i $i=1,2,\ldots,n$, the joint probability function for the sample can be written as:

$$P(X_1;\theta)P(X_2;\theta)\cdots P(X_n;\theta). \tag{11}$$

This product, when viewed as a function of θ for given sample observations, is the likelihood function $L(\theta)$. When maximised with regard to θ, the value of θ is a function of the sample observations and is the maximum likelihood estimator of θ. The likelihood function can be written as:

$$L(\theta) = \prod_{i=1}^{n} P(X_i;\theta) \tag{12}$$

The method of maximum likelihood for estimating an unknown parameter θ selects as a point estimator the value that maximizes the likelihood function (12). When analytical methods are not feasible, the $L(\theta)$ maximizing value of θ can be found by computerized numerical-search procedures. (Netter, Wasserman and Whitmore 1998, 298-301.)

Although Cramer (1986) argues that ML estimators are consistent and asymptotically efficient, and Hall (2005) states that the MLE is the best available classical statistic estimator, there are a number of weaknesses when using the MLE. Hall (2005) lists two particular problems. First, the statistical properties of the MLE are sensitive to the distributional assumption. Arbitrarily chosen distribution can result in biased inferences.

[8] For profound discussion of the maximum likelihood method of estimation, see e.g. Cramer (1986).

Second, in many models, the MLE would be computationally very burdensome. Recently, estimation methods, such as MMLE by Hanson et al. (2004) that try to overcome weaknesses of the MLE, have been developed.

Hanson et al. (2004) show that if the data observations are collected into equally spaced bins, the appropriate estimation method is the *Multinomial Maximum Likelihood Estimation* (MMLE). The result is shown to be independent of the theoretical distribution, since the observed distribution is assumed to be a simulation of i.i.d. random variables. The intuition behind Hanson et al. estimation model is fairly simple. If one can sort the sample data into bins to create a sample distribution of the data, it is possible to search for a parameter vector of a given distribution that returns the best possible fit of a theoretical distribution by minimizing the cumulative difference between each sample bin and corresponding theoretical bin.

Hanson et al. (2004) argue that by using a stochastic chain rule[9], the jump-diffusion process described by (5) can be transformed into a more simple jump-diffusion stochastic differential equation (SDE),

(13) $$d[\ln C_t] = \mu_{ld}dt + \sigma_d dz_t + QdP(t),$$

where μ_{ld}, σ_d and z_t are defined as above. *P(t)* is a standard Poisson jump process with a joint mean and variance λ. For simplicity, only the log-jump-amplitude is defined as *Q= ln(J(Q)+1)*, where *J(Q)* is the Poisson jump-amplitude. If the density of the jump-amplitude component *Q* can be defined as double exponential that is:

(14) $$\phi_Q(q) = \frac{p_1}{\mu_1}\exp\left(\frac{q}{\mu_1}\right)I_{\{q<0\}} + \frac{p_2}{\mu_2}\exp\left(\frac{q}{\mu_2}\right)I_{\{q\geq 0\}},$$

[9] For a comprehensive definition of the stochastic chain rule, see Hanson and Westman 2004, *Applied Stochastic Processes and Control for Jump-Diffusion: Modeling, Analysis and Computation.* SIAM Books, Philadelphia, PA. See URL: http://www.math.uic.edu/~hanson/mcs574/#Text.

where $\mu_1,\mu_2>0$ are one-sided means of exponential distributions and $0<p_1<1$ is the probability of downward jumps. Analogously, $p_2=1-p_1$ is the probability of upward jumps. The set indicator function is $I_{\{S\}}$ for set S.

Q has moments[10]:

(15) $$\mu_j = E_Q[Q] = -p_1\mu_1 + p_2\mu_2$$

(16) $$\sigma_j^2 = Var_Q[Q] = p_1\left((\mu_j+\mu_1)^2+\mu_1^2\right)+p_2\left((\mu_j+\mu_2)^2+\mu_2^2\right).$$

Hanson and Zhu (2004) provide a second order estimation[11] for $[x_1, x_2]$ bin probability distribution of double exponential jump-diffusion; i.e. the probability that an observation is in the given bin $[x_1, x_2]$. If the probability is e.g. 0.1 and the total number of observations in the theoretical sample is 100, there would be 10 observations found from the given bin. The estimator can be written as:

(17) $$\Phi_{dejd}(x_1,x_2) \cong \frac{\sum_{k=0}^{2} p_k(\lambda\Delta t)\Phi_{dedj}^{(k)}(x_1,x_2)}{\sum_{j=0}^{2} p_j(\lambda\Delta t)},$$

for $-\infty < x < \infty$, where

(18) $$\Phi_{dejd}^{(0)}(x_1,x_2) \equiv \Phi_n(x_1,x_2;\mu,\sigma^2),$$

where $\Phi_n(x_1,x_2;\mu,\sigma^2)$ is the normal distribution on the $[x_1, x_2]$, $\mu \equiv \mu_{ld}\Delta t$, $\sigma \equiv \sqrt{\sigma_d^2\Delta t}$, $p_k(\Lambda) = e^{-\Lambda}\Lambda^k/k!$ is the Poisson distribution with the parameter $\Lambda = \lambda\Delta t$, with k jumps and corresponding time increment in years Δt. Subscript *dejd* stands for

[10] The nth moment of a real-valued function $f(x)$ of a real variable about a value c is $\mu_n' = \int_{-\infty}^{\infty}(x-c)^n f(x)dx$

[11] Hanson and Zhu (2004) argue that the additional contribution of a third order approximation is only 1.5% whereas the 2nd order approximation contributes 23% to the first. Therefore, the 2nd order approximation is argued to return satisfactory results. For complete derivation of the model, see Hanson and Westman (2004), Hanson, Westman and Zhu (2004) and Handson and Zhu (2004).

the double exponential jump-diffusion. Detailed definitions of $\Phi^{(1)}$ and $\Phi^{(2)}$ are given in Appendix (A3). The second order estimation approach makes the model significantly lighter to compute compared to some more sophisticated models. Basic moments for log-return increments for log-double-exponential jump distribution defined by Hanson and Zhu (2004) are:

$$M_1^{(dejd)} \equiv E[\Delta[\ln C_t]] = (\mu_{ld} + \lambda\mu_j)\Delta t, \tag{19}$$

$$M_2^{(dejd)} \equiv Var[\Delta[\ln C_t]] = (\sigma_d^2 + \lambda(\sigma_j^2 + \mu_j^2))\Delta t, \tag{20}$$

$$M_3^{(dejd)} \equiv 6(p_2\mu_2^3 - p_1\mu_1^3)\lambda\Delta t, \tag{21}$$

$$M_4^{(dejd)} \equiv 24(p_2\mu_2^4 - p_1\mu_1^4)\lambda\Delta t + 3(\sigma_d^2 + \lambda(\sigma_j^2 + \mu_j^2))^2(\Delta t)^2. \tag{22}$$

Moment-related constraints play a significant role in lowering the computational burden by reducing the number of estimated parameters. Following practical steps describe the main idea of the MMLE method:

Step 1: The sample data should be sorted into *nb* bins in order to find the sample frequency $f_b^{(cf)}$ for each bin. This is a simple frequency distribution, and the sorted data can be graphically presented in a histogram. Hanson uses 100 bins when sorting the data describing S&P500[12] daily closing log-returns from 1992 to 2001.

Step 2: The theoretical frequency of the given distribution model with a parameter vector **x** should be calculated:

$$f_b^{(jd)}(x) \equiv ns \int_{B_b} \phi^{(jd)}(n;x)d\eta, \tag{23}$$

where $\phi^{(jd)}(\eta;x)$ is some jump-diffusion density in η and B_b is the b^{th} bin. Superscripts *cf* and *jd* stand for cash flow and jump-diffusion respectively. Here, the theoretical size of each bin is obtained. Now it is possible to compare the bin frequencies of the observed

[12] The S&P 500 is an index containing the stocks of 500 large, mostly American corporations.

data to the corresponding theoretical frequencies. The optimal fit can be found by minimizing the difference between the distributions.

Step 3: Minimize the objective function

$$(24) \qquad y(x) \equiv -\sum_{b=1}^{nb} \left[f_b^{(cf)} \ln\left(f_b^{(jd)}(x) \right) \right],$$

where the number of the realizations in the theoretical distribution is set equal to the observed number. The objective function looks for a parameter vector **x** that returns the smallest difference between the observed and theoretical distributions.

To reduce the number of estimated parameters $\{\mu_{ld}, \sigma_d^2, \mu_1, \mu_2, p_1, \lambda\}$ to a reasonable amount, the first and second theoretical moments are set to be equal with the corresponding empirical mean and variance,

$$(25) \qquad M_1^{(dejd)} = M_1^{(cf)}$$

$$(26) \qquad M_2^{(dejd)} = M_2^{(cf)}.$$

To eliminate further the diffusion parameters μ_{ld} and σ_d, the following definitions are set:

$$(27) \qquad \mu_{jd} = \left(M_1^{(cf)} - \mu_j \lambda \Delta t \right) / \Delta t$$

$$(28) \qquad \sigma_d^2 = \max\left[\left(M_2^{(cf)} - \left(\sigma_j^2 + \mu_j^2 \right) \lambda \Delta t \right) / \Delta t, \varepsilon \right].$$

Sufficiently small $\varepsilon > 0$ is set to control the positivity. With the constraints described above, the original six parameters can be reduced to four $x = \{\mu_1, \mu_2, p_1, \lambda\}$. To find the optimal parameter vector, the MATLAB mathematical applications' unconstrained search function `fminsearch` is used. The complete MATLAB code for the parameter estimation is presented in Appendix (A5).

3 Impulse control models

This chapter introduces the basic principles of impulse control models and presents the specific algorithm to model optimal cash management policy originally introduced by Bar-Ilan, et al. (2004). In their model, Bar-Ilan et al. define the cost parameters that influence optimal management of a firm's cash position. First, holding liquid assets bears a holding cost h that represent the opportunity cost of all the foregone investment opportunities. Adjusting the level of liquid holdings carries a cost with two components. The fixed cost component K_i is independent of the amount of the adjustment. Payments such as standard bank transfers fees can be seen as a fixed cost component. On the other hand, the variable component k_i depends on the amount transferred. A percentage-based broker's fee can be used as an example of a variable cost component. An optimal cash management policy can be obtained by minimising the discounted future costs accumulating from liquid holdings.

The underlying idea of the impulse control models that use trigger-target rules to obtain optimal cash management policy is straightforward. The optimal transfer rule is often described in a simple form, where the amount of the money transferred at the beginning of the period $n = 1,2,\ldots$ is:

(29)
$$f(s_n) = \begin{cases} s_l - s_n, & \mathrm{s_n} \le S_l \\ \mathrm{s_n}, & S_d < \mathrm{s_n} < S_u \\ s_n - s_u, & \mathrm{s_n} \ge S_u \end{cases}.$$

The rule indicates that the cash level is unchanged if it is within the limits S_u and S_d and otherwise reset to s_u and s_d, respectively. *Figure 3* below is a graphical presentation of a typical impulse-controlled sample path. Here individual dots represent the level where the process would have been at the moment when it was impulse-controlled back to the upper or lower target level.

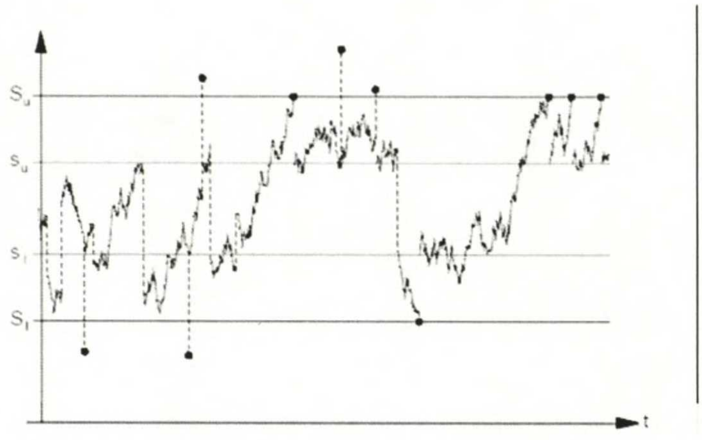


Figure 3: Typical impulse controlled sample path (Bar-Ilan et al. 2004, 1017).

Under the assumption of BM as an underlying stochastic process, Harrison, Sellke and Taylor (1983) show that there exists an optimal, expected discounted total cost minimising, *impulse control band* (ICB) policy. Furthermore, Bar-Ilan et al. (2004) argue that for given trigger values S_u and S_l, if the targets s_u and s_l can be freely chosen, there is such an s^* within the range (S_l, S_u) that the optimal choice of target(s) is $s^*=s_u=s_l$.

When relaxing some of the assumptions of the original Miller-Orr model mainly by characterizing the underlying stochastic process as jump-diffusion, Bar-Ilan et al. (2004) argue that the objective is to choose an ICB policy that minimizes the following expected discounted cost:

(30)

$$R_x(\beta) = R_x(\beta; S, s_u, s_l) =$$
$$E_x\Big(\int_0^\infty e^{-\beta t} h(W(t))dt + k_u \int_0^\infty e^{-\beta t} dU(t) + k_l \int_0^\infty e^{-\beta t} dL(t) + K_u \int_0^\infty e^{-\beta t} dI_U(t) + K_l \int_0^\infty e^{-\beta t} dI_L(t) \Big).$$

This fairly complex-looking equation is simply the expected sum of all discounted future costs presented in a continuous time framework. Here, β denotes the discount interest rate; k_u, K_u, k_l, K_l are transaction cost constants where the capital letter denotes fixed costs of a transaction and the lower case letter variable costs proportional to the amount transferred. Subscripts u and l denote upward and downward adjustments respectively; $h(\cdot)$ is the holding cost rate function corresponding to the cost of holding cash money; $U(t)$ and $L(t)$ are the cumulative sums of downward and upward adjustments, respectively, up to time t; W indicates the ICB-controlled cash flow process; dU and dL are the jump size of U and L, and I_U and I_L are the number of jumps. Without a loss of generality, the lower trigger level S_l has been set to zero (Bar-Ilan et al. 2004, 1017). In other words, the discounted amount and number of all future cash increases (reductions) are counted, multiplied by the cost constant and summed together with the discounted cost of cash holdings.

If the underlying stochastic process is a double exponential jump process and the process parameters are known, the optimal ICB policy can be presented explicitly. Bar-Ilan et al. (2004) provide a method of presenting (30) in terms of additive components that can be used to calculate the optimal policy. Practical steps for an empirical analysis presented by Bar-Illan et al. are introduced in the rest of the chapter. The corresponding MATLAB programming code is reported in Appendix (A6). The following algorithm is in its whole referring to the work of Bar-Ilan, Perry and Stadje (2004).

At first, one should specify from the underlying stochastic jump-diffusion process (i) the drift and the variance of BM, μ and σ, (ii) the intensity and mean of the upward (downward) jumps λ (ξ) and η^{-1} (v^{-1}) respectively, (iii) and the discount factor β. The holding costs h_0, k_u, K_u, k_l, K_l described above should also be determined. Second, one should find four $\alpha_1(\beta) > \alpha_2(\beta) > 0 > \alpha_3(\beta) > \alpha_4(\beta)$ roots for the equation (31) defining the parameters for the double exponential jump diffusion:

$$(31) \qquad \beta = \frac{\sigma^2\alpha^2}{2} - \mu\alpha - \frac{\lambda\alpha}{v+\alpha} + \frac{\eta\alpha}{\xi-\alpha}.$$

The additive components $\phi_x^i(\beta)$ can thereafter be obtained by solving:

(32) $$\left(\phi_x^0(\beta),\phi_x^1(\beta),\phi_x^2(\beta),\phi_x^3(\beta)\right)' = A(\beta)^{-1}\left(e^{-\alpha_1(\beta)x},e^{-\alpha_2(\beta)x},e^{-\alpha_3(\beta)x},e^{-\alpha_4(\beta)x}\right)',$$

where ' denotes transposition of the vector and -1 inversion of the matrix $A(\beta)$ defined by (33) below. The inversion is performed by using the MATLAB `inv()` function.

(33) $$A(\beta)=\begin{pmatrix} 1 & \dfrac{\xi}{\xi-\alpha_1(\beta)} & e^{-\alpha_1(\beta)S} & \dfrac{\nu}{\nu+\alpha_1(\beta)}e^{-\alpha_1(\beta)S} \\ 1 & \dfrac{\xi}{\xi-\alpha_2(\beta)} & e^{-\alpha_2(\beta)S} & \dfrac{\nu}{\nu+\alpha_2(\beta)}e^{-\alpha_2(\beta)S} \\ 1 & \dfrac{\xi}{\xi-\alpha_3(\beta)} & e^{-\alpha_3(\beta)S} & \dfrac{\nu}{\nu+\alpha_3(\beta)}e^{-\alpha_3(\beta)S} \\ 1 & \dfrac{\xi}{\xi-\alpha_4(\beta)} & e^{-\alpha_4(\beta)S} & \dfrac{\nu}{\nu+\alpha_4(\beta)}e^{-\alpha_4(\beta)S} \end{pmatrix}.$$

Using the additive components from (32), the trigger increases (decreases) and the cash increase (reduction) functionals can be obtained.

Upper trigger cash reduction functional is:

(34) $$E_2 = \frac{\Phi_x(\beta)\left[\phi_x^2(0)(S-s_u)+\phi_x^3(0)(S-s_u+\frac{1}{\nu})\right]}{1-\Phi_{su}(\beta)\left[\phi_{su}^2(0)(S-s_u)+\phi_{su}^3(0)(S-s_u+\frac{1}{\nu})\right]}.$$

Lower trigger cash increase functional is:

(35) $$E_3 = \frac{\Theta_x(\beta)\left[\phi_x^0(0)s_l + \phi_x^1(0)(s_l + \frac{1}{\xi})\right]}{1 - \Theta_{sl}(\beta)\left[\phi_{sl}^0(0)s_l + \phi_{sl}^1(0)(s_l + \frac{1}{\xi})\right]}.$$

Counting function for the cash reductions is:

(36) $$E_4 = \frac{\Phi_{sl}(\beta)}{1 - \Phi_{su}(\beta)}.$$

Counting function for the cash increases is:

(37) $$E_5 = \frac{\Theta_{sl}(\beta)}{1 - \Theta_{sl}(\beta)},$$

where $x=sl$ and $x=su$ respectively,

(38) $$\Theta_x(\beta) = \frac{\phi_x^0(\beta) + \phi_x^1(\beta)}{1 - \phi_x^2(\beta) - \phi_x^3(\beta)}$$

(39) $$\Phi_x(\beta) = \frac{\phi_x^2(\beta) + \phi_x^3(\beta)}{1 - \phi_x^0(\beta) - \phi_x^1(\beta)}.$$

Furthermore, *the expected holding cost is:*

(40) $$E_1 = \frac{H_x(\beta)}{1 - \Theta_x(\beta)},$$

where

(41) $$H_x(\beta) = -\frac{\partial}{\partial\alpha}\varepsilon_x(\alpha,\beta)\big|_{\alpha=0},$$

and

(42) $$\Theta_x(\beta) = 1 - \beta\varepsilon_x(0,\beta),$$

where $\varepsilon_x(\alpha,\beta)$ represents a discounted holding cost $E_x = \left(\int_0^T e^{-\alpha W(t) - \beta t} dt \right)$.

With the functions set above, the linear combination of the expected discounted cost is

(43) $$R_x(S,s_u,s_l) = h_0E_1 + k_uE_2 + k_lE_3 + K_uE_4 + K_lE_5$$

Keeping in mind that the lower trigger was set to zero, the cost functional (43) can be used when seeking parameters that minimize the expected cost.

IV Data and time series properties

This section describes the data used in this study together with a discussion about the reasoning behind the choices of specific methods. First, the general properties of the data are introduced. In the second chapter, the time series properties of the data are discussed with some arguments about the underlying distribution. In chapter IV 3, the parameter estimates for the underlying stochastic process are reported. The section concludes with some analysis of the feasibility of the obtained results.

1 Data

The firm-specific data of this study has been collected from the data warehouse (DW) of the case company. The data consists of the net of daily accounts payable (AP) and accounts receivable (AR) for the time period from 1.1.2005 to 31.12.2006, i.e. 564 recorded observations. This will lead to 563 changes in the level of the cash holdings (return).[13] The return is measured in natural logarithms $x_r = ln(C_t/C_{t-1})$. The cash and cash equivalent information from the balance sheet of the case company has been used as the starting level for the liquid holdings on 1.1.2005. The information has been collected from the annual report of the case company for the year 2005. The analysis ignores the effect of weekends and banking holidays. The fact that days when trading is not possible has an effect on optimal cash policy is recognized. However, this effect is considered insignificant for the analysis. All figurers reported in this work have been subject to monotonic transformation in order to assure integrity of the case company.

Naturally, the AP and AR transactions do not give a complete picture of the financial position of the company. Many important cost items are excluded from the study e.g. salaries, taxes, dividends etc. However, the main purpose of this study is to analyse the cash flow risk that the case company faces, and a fair argument is that the cash flow

[13] For simplicity only the "return" notation is used to describe the change in the level of cash holdings.

from the operations pose the majority of that risk. The cost items left out of the study are also mostly predictable, such as salaries. One might of course argue that the AP is also well known information by any company, thus the only uncertainty that any company faces in the short run is the AR. However, within the industry that the case company operates in, there are some payment conventions that make the predictability of the AP items more difficult than would be in most conventional industries. Opportunistic behaviour by the company can also lead to unpredictable cash flows. Keeping in mind the points mentioned above, it is reasonable to conclude that the net of AP and AR is a good representation of the cash flow risk that the case company faces.

Figure 4 below is a representation of the cash flow data. Consecutive realisations are added up to obtain a cumulative time series from individual daily observations. The fact that the data consists of only the net of AP and AR results in a strong upward trend. Naturally, this is not realistic. Fixed items and other expenses left out of the analysis would eventually level off the trend. The actual realized trend of the company could be defined by comparing the balance sheet items of the case company in consecutive years.

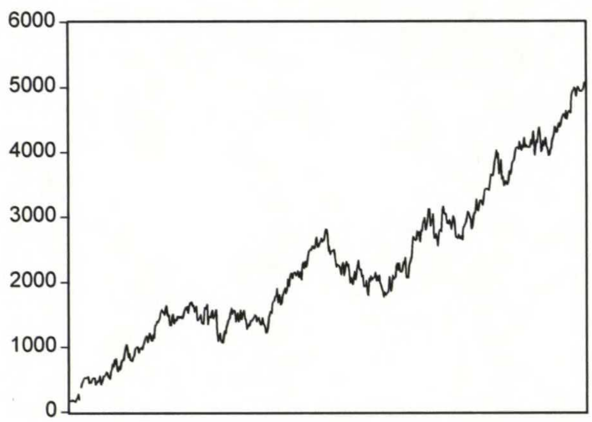
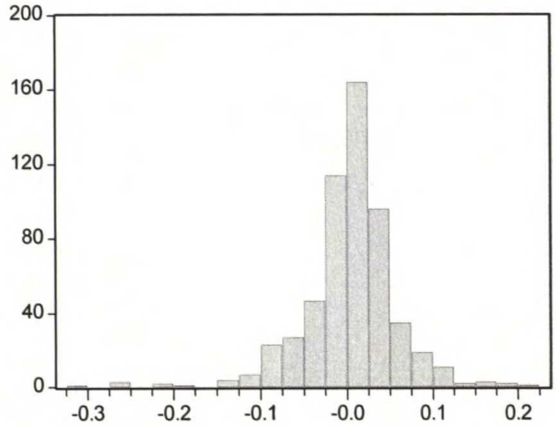


Figure 4: Realization of cash flow presented as a cumulative (daily) time series.

2 Time series properties

After understanding the general properties of the data, it is now possible to seek a stochastic process underlying the cash flow. Tapiero (1998) argues that finding an appropriate stochastic process that is a good representation of the evolution of a time series or a stochastic environment is still an unsolved issue. Nevertheless, Tapiero identifies a list of three requirements that should be followed when constructing a stochastic process. First, one should identify the evolutionary structure of the process. Second, the formal characterization that formulates such evolution should be found; and finally, a model should be constructed in terms of disposable information. Steps introduced by Tapiero are followed in the rest of this chapter.

First, visually analysing the time series (*Figure 4*) does not clearly indicate the underlying stochastic process. Therefore, the possibility of GBM is at first looked into. This can be done by analysing the lognormal property of the time series. *Figure 5* below is a histogram representation of the cash flow when the return is measured in logs. The relevant parameters are generated by the *EViews* econometric application. One observation (12.1.2005; 0.64) has determined to be an outlier and limited outside the sample.



Series: LCASH	
IF LCASH < 0.5	
Observations 562	
Mean	0.004673
Median	0.007863
Maximum	0.213143
Minimum	-0.316119
Std. Dev.	0.055175
Skewness	-0.983339
Kurtosis	8.481911

Figure 5: A histogram representation of the cash flow measured in logs

Normal distribution of the log-returns would indicate a Brownian Motion as an underlying stochastic process. Analysis of the histogram in *Figure 5* exposes thick tails and peakedness of the distribution indicating a stochastic process that does not follow GBM. Negative skewness (-0.98) and leptokurtic behaviour (kurtosis 8.48) supports the initial visual observation. Merton (1976), Kou (2002), Chacko and Viceira (2003), Hanson and Zhu (2004) and Ramezani and Zeng (2006) among others argue that jump-diffusion models could describe negative skewness and leptokurtic behaviour in a stochastic process.

The standard jump-diffusion model by Merton (1976) consists of components of a linear drift, Brownian motion and a compound Poisson process, i.e. the evolution of the time series follows a random walk process (with a trend) with occasional jumps up (and down). This type of stochastic process is common when characterising the evolution of a stock price. The stock market reacts to unexpected news and - by efficient market hypothesis - the price of the share jumps immediately to the new level that reflects all the available information (see e.g. Brealey & Myers, 2003). The cash flow of a corporation can be subject to similar jumps. A firm can have some relatively large clients whose transactions have a significant jump effect on the cash flow. Cash flow jumps can also be the result of opportunistic behaviour of the firm deriving from a favourable commodity price development. Hodrick-Prescott's (1997) decomposition of the daily cash flow observations (λ=13,322,500) gives a better understanding of the jumps in the cash flow data. The "Cycle" line in *Figure 6* below is a realisation of the cash flow process without the trend component. This representation magnifies the jumps up (down) that are difficult to observe in the original representation of the data. Jumps can now be easily identified as vertical shifts in the "Cycle" line. The black arrow in the picture points out one of the downward jumps.

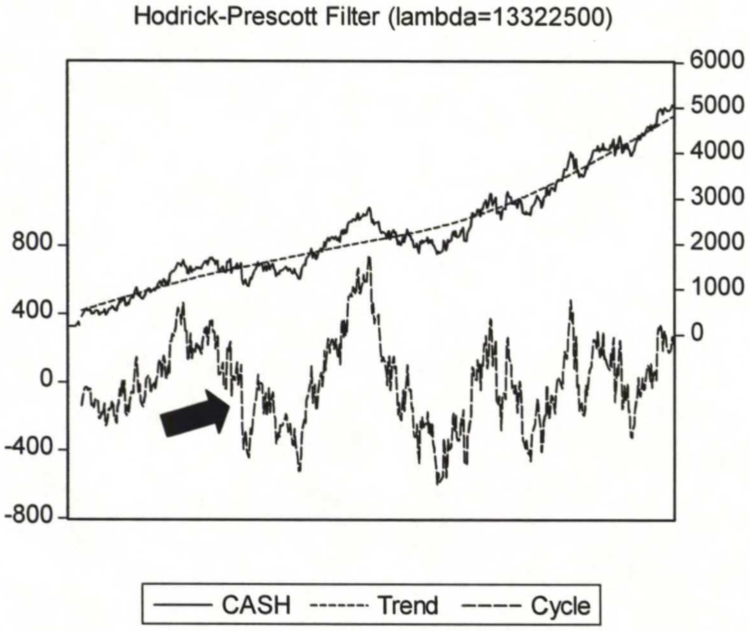


Figure 6: Hodrick-Prescott decomposition of the daily cash flow data

Merton's (1976) model has become the most important representation of the jump-diffusion (Ramezani and Zeng 1999, 3). However, the lognormal jump component proposed by Merton may not be a good description of a corporate cash flow process. The general definition of a majority of corporate business activities, i.e. buy raw material in large lot sizes and sell finished goods in small batches, generates asymmetric jumps. The limited liability models, such as the one described by Ramezani and Zeng (1999), account for asymmetries in jumps. However, a corporate cash flow can in fact witness jumps that exceed the boundaries set by the limited liability models. Corporations can have overdraft limits in their bank accounts that allow down jumps that exceed 100%.

In their generalized impulse control model of cash management, Bar-Ilan et al. (2004) build on the assumption of a double exponential jump-diffusion process where the jump component follows asymmetric exponential distributions. For the cash flow data, this would allow the scale of AP jumps to differ from the scale of AR jumps and jump sizes that could exceed 100%. This is a reasonable assumption, considering the nature of the corporate cash flow in general. It is also good to recognise the fact that the DEJD model does not rule out the possibility of symmetric jumps nor does the model force jumps beyond the limited liability. One should also notice that the model reduces to a regular BM in case of zero jumps. Therefore, the DEJD model could be argued to be a generalized version of Merton's (1976) original representation.

In the light of the initial analyses of the data and the discussion above, the parameters of the cash flow process in this work are estimated assuming the underlying stochastic process to follow the double exponential jump-diffusion. The possibility of a different type of stochastic process as a representation of the underlying process is also recognised. However, as shown later in this section, the DEJD model is a good representation of the underlying process for the sample data. Thus, the DEJD approach is chosen with confidence.

Ramesani and Zeng (1999) list a number of methods for estimating the jump-diffusion models, such as maximum likelihood estimation, method of moments and its variants including cumulant-matching generalized method of moments, and simulated method of moments. Chacko and Viceira (2003) add to the list discretization, Edgeworth expansion, non-parametric approximation to the density function and the generation of moment restrictions through random sampling. Most of these methods require computational ability beyond the scope of this work. In this work, the multinomial maximum likelihood estimation (MMLE) by Hanson et al. (2004), also Hanson and Zhu (2004), is used. Hanson argues that the method significantly reduces the computational burden of the estimation. The method is also, despite the complex proof of the model, fairly simple to implement and thus suitable for a study of this scale. The general principles of the model were introduced in chapter III 2. The following chapter introduces the practical steps in the process for choosing the underlying distribution, estimating the parameters and reporting the results of the estimation.

3 Parameter estimates

In this chapter, parameter estimates of the underlying stochastic process are reported and analysed. The MMLE method (Hanson et al. 2004) for finding theoretical bin probability distribution is used to find an optimal fit of the data. First, the superiority of the jump diffusion over a simple GBM is shown. Second, the parameter estimates are reported. The chapter concludes with some critical discussion about the choice of the model.

Figure 7 below describes the frequency distribution of log returns of a time series in a histogram representation. Neter, Wasserman and Whitmore (1988) argue that the number of classes in a frequency distribution has to be determined by experimentation. If the number of classes is too large, the representation loses its effectiveness of summarizing the data. On the other hand, too few classes may condense the information too much. The sample data has been sorted to 70 bins. The width of each bin is 0.01. This choice of bin range is believed to return a reliable representation of the data. Hereinafter, references to "the model" or "the method" refer to the one described by Hanson et al. (2004), unless otherwise mentioned.

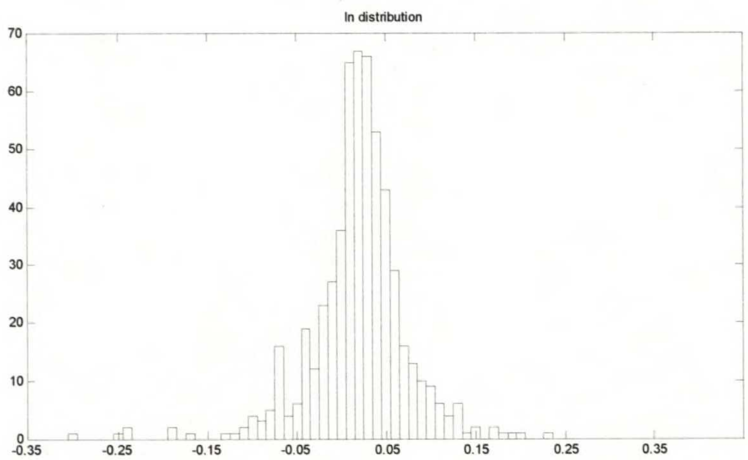


Figure 7: Histogram representation of the log returns sorted to 70 bins of equal width

When applying the method to find an optimal fit of a normal distribution to the time series data, the theoretical distribution illustrated by the dotted line in the *Figure 8* below is found. The corresponding value of the minimized objective function (24) is -1799.00. The mean and standard deviation obtained are 0.0042 and 0.0551 respectively. The unconstrained nonlinear optimization function `fminsearch` of the MATLAB mathematical application was used for the optimization process. The *Nelder-Mead* simplex direct search algorithm provided by MATLAB was used in the optimization. The starting points for the search of mean and standard deviation were set to 0.1 each.

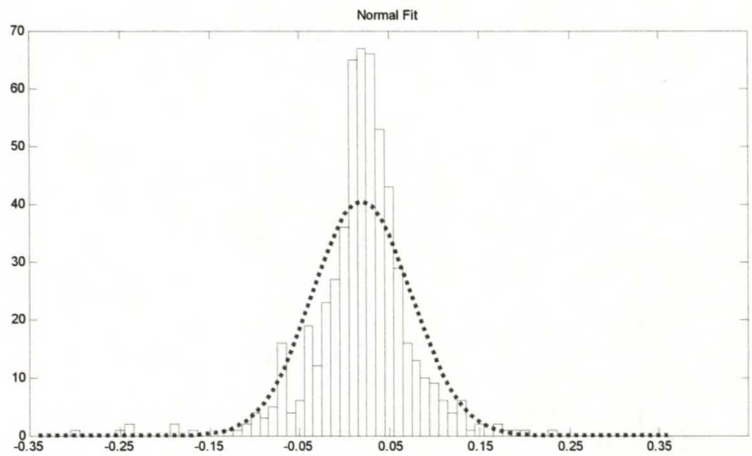


Figure 8: Fit of the normal distribution

The `fmisearch` function of MATLAB was also used when seeking the best possible fit of the DEJD model. The initial estimates for the number of jumps, probability of a negative jump, negative jump mean and positive jump mean were set to 1; 0.5; 0.1 and 0.1 respectively. *Figure 9* below is the graphical presentation of the optimal fit of the DEJD model. Again, the dotted line represents the theoretical distribution compared to the bar representation of the observed data. The visual illustration provides a clear difference between the two models. As one can easily see by comparing *Figure 8* and

Figure 9, the fit of DEJD is significantly better than that of GBM. The smaller value of the objective function (24) -1873.4 (-1799.0) also indicates a better fit. The full MATLAB code for both of the models is presented in the Appendices (A4) and (A5).

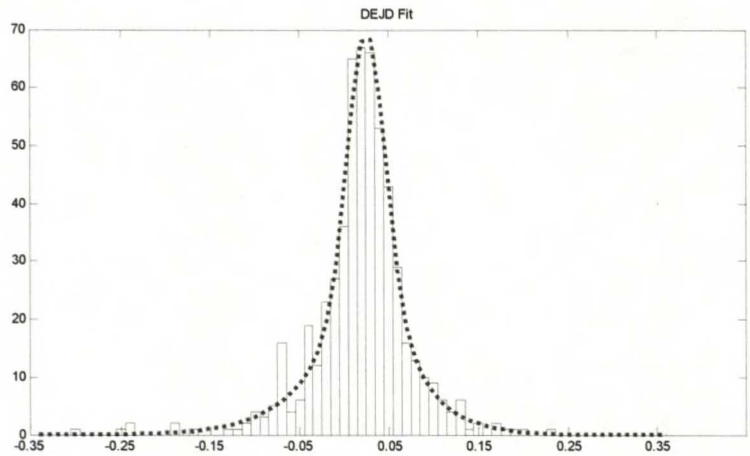


Figure 9: Fit of the DEJD distribution

It is now shown that the DEJD compared to the GBM is a superior model when characterizing the underlying stochastic process of the time series data. Keeping the earlier discussion of the correct choice of the jump parameter distribution in mind, the parameter estimates can now be presented with confidence. The initial values of mean 0.004673 and standard deviation of 0.055175 calculated from the sample data were used to set the constraints introduced by Hanson et al. (2004). The time increment in years Δt was set to 0.003546 to correspond to 564 recorded observations in a two-year time interval. The optimization procedure presented in Appendix (A5) was run using the `fminsearch` function of MATLAB. Four optimal parameters were found.

Table 5: Optimal DEJD parameters

λ	302.6395	*number of jumps*
p_1	0.5204	*probability of a downward jump*
μ_1	0.0408	*negative jump mean*
μ_2	0.0333	*positive jump mean*

When analyzing the results, one should take a new look at the Hodrick-Prescott (1997) filtered data. *Figure 10* consists of only a cycle line that is a representation of the time series without the trend component. The horizontal line is set to zero.

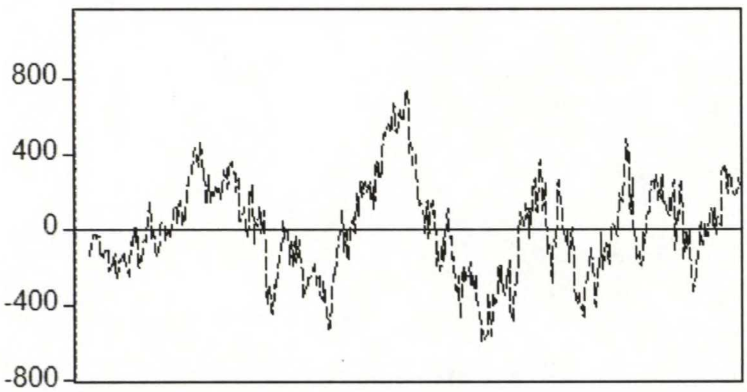


Figure 10: Hodrick-Prescott filtered time series data without a trend component

The parameter estimates indicate that the number of jumps within the observed data is 302.6. That is 53.7% of the observed changes of the daily cash holding. Therefore, more than half of the realizations would be explained by the jump component. *Figure 10* above supports this view. One can easily see that even the large jumps that can be visually detected are frequently observed. The large number of jumps raises a question of the CPP as a sole contributor of the underlying process. This possibility is nevertheless implicitly built in the model. If the data could be optimally described as a CPP, the number of jumps would be equal or close to the number of observed realizations. And vice versa: if the BM were the sole contributor, the number of jumps in optimum would be zero.

The considerably large number of jumps up and down together with an almost equal amount of small daily cash holdings indicate that the business operations could be characterized with two different categories i.e. large and small-scale operations. This view is consistent with the business environment of the case company. The operative business is on the one hand selling smaller amounts of goods to the end users but on the other hand handling large quantity deals with other global operators. Here, the diffusion component describes the former type of operations and the jump component the latter type.

The optimization results also indicate a different type of behavior of inflows and outflows of money. Both the larger probability of downward jumps 0.52 (0.48) and the larger mean of the negative jumps 0.04 (0.03) indicate that jumps appear more frequently and are larger with outflows of money. In accordance with this observation is the fact that buying the raw material is dominated by larger quantity deals. This result is virtually impossible to visually detect from the *Figure 10*. Nevertheless, the general business environment of the majority of businesses supports this result. Raw materials are brought in large quantities and finished goods sold in small batches. To conclude the analysis, it can be argued with a confidence that the obtained results are a good representation of the underlying stochastic process and are in line with the real business operations of the case company.

To complete the analysis of the optimal cash management model, the parameters required for the trigger-target model are derived from the results obtained above. Using these parameters, the optimal cash management policy for the case company is set in the following chapter. Using the definitions set by Hanson et al. (2004), the DEJD parameters required for the impulse control model are:

Table 6: Parameters for the impulse control model

Drift of the BM	μ	2.91
Variance of the BM	σ^2	0.11
Intensity of upward jumps	γ	0.51
Intensity of downward jumps	ν	0.56
Mean of upward jumps	η^{-1}	0.0333
Mean of downward jumps	ξ^{-1}	0.0408

From the parameters presented in *Table 6* above, the means η^{-1} and ξ^{-1} are withdrawn directly from the optimization results. The drift, variance and intensities are derived using the argumentation by Hanson et al. (2004). μ_j and σ_j are defined as described in (15) and (16), λ is the total number of jumps and Δt is the time increment in years. The intensity of jumps for up and down $x = \nu$ and $x = \gamma$ respectively are defined as

$$x = p_i \, \Delta t \, \lambda, \tag{44}$$

where p_i is the corresponding probability of a jump i=1 for jumps down and i=2 for jumps up.

The search for the parameters of the underlying stochastic process is now ready. In conclusion, the parameter estimates appear to be consistent with the initial analysis of the business environment and can therefore be used when seeking the optimal short-term cash management policy for the case company. However, some remarks of caution should be mentioned here. The following cash management policy analysis leans heavily on the results reported above. As discussed earlier, the choice of the underlying stochastic process is still an unsolved issue. Also, the possibility of an incorrect choice of the process would return biased conclusions about the optimal cash management policy. Beside the choice of the underlying process, one should also understand that the results obtained here represent a still photograph of the situation at the end of 2006.

This means that the cash management policy set in the following section assumes that the cash flow process will continue to develop in future in the same manner as in the past. With these arguments in mind, it is now possible to proceed with the optimal trigger-target policy. The optimal policy is set and the corresponding results reported together with a sensitivity analysis in the following section.

V The impulse control policy

In this section, the parameter estimates of the double exponential jump diffusion together with company-specific parameters are imputed to the Generalized Impulse Control Model of Cash Management by Bar-Ilan et al. (2004). As a result, an optimal cash management policy for the firm is obtained. First, the reasoning behind the choices of the company-specific cost parameters is argued. After this, the optimal cash management policy is reported, followed by a sensitivity analysis of the key parameters. The chapter concludes with a discussion of the advantages of the model and some pitfalls that should be considered.

1 Company-specific parameters

When setting the cost parameters, some generalizations are made. First, the holding cost h_0 is set to 1. The proportional transaction costs k_u and k_l are set to 1.2 each. This indicates a transaction cost that is 1.2 times larger than the holding cost. To ensure that the assumption is adequate, the model is tested by increasing all cost constants by an equal percentage (10%) and regenerating the results. It is found that the optimal cash policy remains unchanged. Regardless, the definition of the cost constants requires some further discussion.

For a corporation that uses floating interest rates such as *euribor*[14] as the underlying reference rate for its financing, the fixed multiplier between holding cost and transaction cost is not adequate. As the reference rate fluctuates, the multiplier should also fluctuate. In fixed interest rates, the problem does not arise. Therefore, financial instruments such as interest rate swaps[15] could be used to trade floating interest rates to fixed ones. On the other hand, it is later shown that in a relevant region, the proportional

[14] European reference rate set daily by the European Central Bank.

[15] Financial swap is an arrangement between two parties in order to change cash flows on agreed terms over an agreed period (Nordea Bank Danmark A/S, 2004).

transaction-cost multiplier does not have a significant effect on the obtained trigger and target levels.

The proportional cost of acquiring new funds by selling securities and investing excess funds are here set equal. One may argue that this is not a realistic view of the financial markets because the cost of borrowing differs from the cost of lending. On the other hand, for a leveraged company this equality can hold. Brealey and Myers (2003) argue that the optimal debt for a company is greater than zero. It is common for large corporations to hold some amount of debt on their balance sheet. Therefore, what is in the model described as requiring additional funds by selling securities can be defined here as acquiring funds by obtaining additional debt. Likewise, what is defined as investing excess funds to securities is here understood as instalments of debt obligations. This is consistent with the case company that has a significant amount of debt in the balance sheet (gearing ratio 34.4%)[16].

Therefore, for a leveraged company, the holding cost and the proportional transaction costs can be defined as follows. The holding cost h_0 is the cost that the company is paying for a liquid source of funding e.g. a line of credit or commercial paper (CP) program. However, the liquid source is in most cases limited in size and at some point additional funding has to be obtained from other sources, e.g. a bank loan. The withdrawal and instalment of the loan comes with a cost (proportional and fixed). The proportional cost is the difference between the cost of liquid funding and the bank loan. Likewise, the proportional cost of decreasing the amount of cash holdings should be understood as the difference between the cost of a bank loan and a foregone investment opportunity.

From the viewpoint of a shareholder, the company can use excess funds to pay back a loan, thus saving in loan costs. On the other hand, the company could reinvest the money in order to gain profit for the shareholder. Therefore, if a company's expected rate of return for a shareholder is larger than its savings from the debt instalment, the

[16] The gearing ratio is collected from the company's annual report for the financial year 2006.

decreasing of the cash level by an instalment of a debt bears a proportional cost as the return for the shareholder is smaller than it would be when investing to the business. One could argue that in this case, the bank loan should never be paid back. However, bank loans come with some covenants and maturity attached; therefore, the amount of loans cannot be increased over some predetermined level and must be paid back at some point in the future.

To simplify this fairly complex cost definition, the proportional costs are set equal. The complete cost structure can be described as follows. The cost of holding liquid assets is set to 1. The cost of debt is 1.2 times the cost of holding liquid assets, and the shareholder's expected rate of return is 1.2 times the cost of debt. The fixed cost component K_u and K_l are set to 1 in order to accommodate the effect of the fixed component on the model. The sensitivity analysis below reveals the significance of the fixed cost component.

The discount factor β is set to 0.05 (5%). The discount factor has a significant effect on valuation methods, such as the discounted cash flow method (DCF). Thus, it has a direct effect on the value of the corporation and its publicly traded shares. The choice of the correct discount factor is widely discussed issue in financial literature. The basic finance textbooks, such as Ross, Westerfield and Jaffe (2003); Brealey, and Myers (2003), argue that the weighed average cost of capital (WACC) is the correct approach for choosing a company-specific discount factor. The simplified definition for the weighed average cost of capital is

$$WACC = expected\ cost\ of\ debt * debt/(debt + equity) + expected\ cost\ of\ equity * equity/(debt + equity)$$

Therefore, the WACC should by definition lie between the cost of debt and the cost of equity (see e.g. Brealey and Myers 2003, 231). The cost of equity is defined as the expected rate of return of the shareholders. It should be self-explanatory that the expected rate of return for shareholders is greater than the cost of debt. The shareholders bear a risk that is greater than the debtors' risk because of the debtors' privileges in the case of default.

All the parameters required for the impulse control model are now set. The jump-diffusion parameters were reported in the previous chapter and the cost parameters together with the discount factor were defined above. The optimal trigger-target search can now be obtained by minimizing the objective function (43).

2 Solution

This chapter presents the results of optimal impulse control policy. First, the optimisation results are reported and interpreted. The sensitivity with regard to the analysis of the model with regard to the relevant parameters is performed in the following chapter.

In line with Bar-Ilan et al. (2004), the results of this work indicate that the target levels are equal when they are freely chosen within the trigger interval of the triggers in the optimum upper and lower targets. When the lower trigger is set to zero $S_l=0$ and the MATLAB `fmincon` minimum search function is used with the following constraints:

Table 7: MATLAB minimum search function constraints

$s_u - s_l$	≥ 0	*upper target is larger than or equal to lower target*
$S_u - s_u$	≥ 0	*upper trigger is larger than or equal to upper target*
s_l	≥ 0	*lower target is greater than or equal to zero*
s_u	≥ 0	*upper target is greater than or equal to zero*

If the MATLAB search parameters are set to $s_l = w(1)$, $s_u = w(2)$, $S_u = w(3)$ and the constraints rearranged to equivalent "smaller than" form, the constraint can be written in matrix form A*w ≤ b. The conditional search returns the following objective function-minimizing results:

Table 8: Optimal trigger and target levels

s_l	0,4331
s_u	0,4331
S_u	1,3635
R(·)	18,1303

Noteworthily, the MATLAB application reports that the constraints have become active while searching for the optimum. Therefore, it is relevant to examine the behaviour of the objective function. The three-dimensional presentation of the objective function (*Figure 11*) indicates that when $s^*=s_u=s_l$, the reported result is at a least the local minimum of the objective function. In *Figure 11*, the *y*-axis represents realizations of S_u in an interval [1,2] and the *x*-axis realizations s^* in an interval [0.2,0.6]. The *z*-axis represents the corresponding value of the objective function *R(·)* for any given combination of trigger and target levels within the given intervals. The graph clearly indicates that the obtained result is at least the local minimum of the objective function *R(·)*.

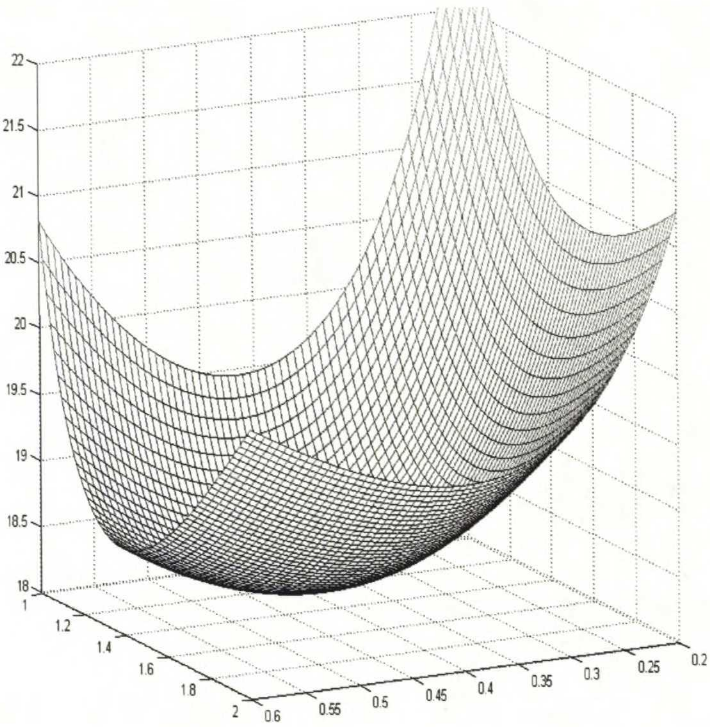


Figure 11: Realizations of the objective function

The interpretation of the result is as follows. The stochastic process underlying the cash flow was presented in relative terms i.e. logs. Therefore, the optimal ICB policy returns also in relative terms. Straightforward intuition behind the reasoning is that any company that has the same relative changes in the cash levels will return the same jump-process parameters. The lower trigger level is set to zero. This corresponds to a firm's choice of the minimum level of cash holdings. The optimal target level s^* is 0.4331, i.e. the optimal target is 1.43 times the lower trigger set by the firm. The optimal upper trigger S_u is 1.3635, i.e. 2.36 times the lower trigger.

3 Sensitivity analysis

Sensitivity analysis of some parameters defined above is also in place. As discussed above, the choice of cost constants and discount factors is not as straightforward as one could desire. It is therefore reasonable to analyse what is the effect of optional choices of the optimal ICB policy parameters. First, the model sensitivity with regard to discount factor β is analysed. *Figure 12* indicates that the objective function (*y-axes*) is a decreasing function of β (*x-axes*). This is quite self-explanatory. When the discount factor increases, the value of the future cash holdings degreases. Thus, the discounted total cost must also decrease.

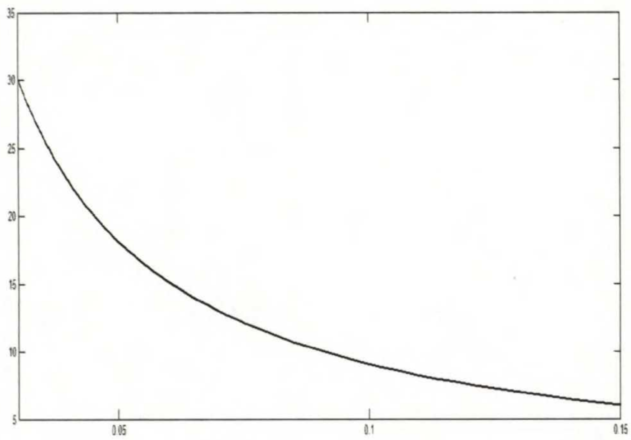


Figure 12: Objective function sensitivity with regard to discount factor β

The effects on the optimal policy are not quite so evident. In order to analyse the effects on the optimal policy, different values of β are set and the optimal policy recalculated. The results are reported in *Table 9*.

Table 9: Sensitivity of the optimal ICB policy with regard to the discount factor β

β	s^*	S_u	$R(\cdot)$
0.05	0.4331	1.3635	18.1303
0.06	0.4330	1.3725	15.1299
0.07	0.4334	1.3818	12.9789
0.08	0.4346	1.3915	11.3597
0.09	0.4362	1.4011	10.0957
0.10	0.4381	1.4108	9.0808
0.11	0.4405	1.4205	8.2476
0.12	0.4432	1.4303	7.5508

Plotting the optimal trigger and target levels for the given values of beta reveal a somewhat interesting result. *Figure 13* below indicates that the upper trigger S_u (right scale) increases in a linear fashion with β, while the optimal target s^* (left scale) increases in an exponential manner.

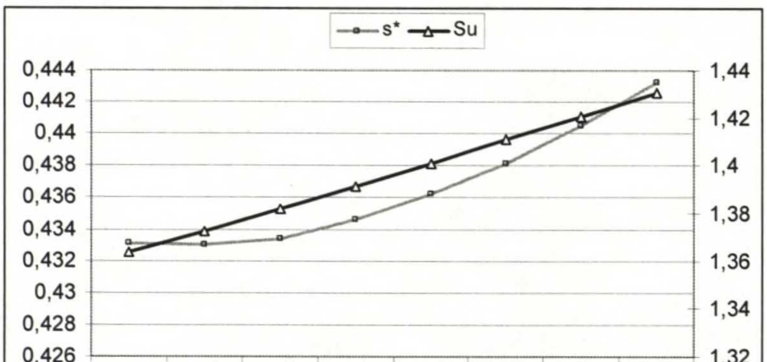


Figure 13: Optimal trigger and target levels with regard to discount factor β

This result indicates that as the β increases - i.e. value of future cash flows decreases - the company will be better off by having larger cash holdings. Considering how the cost structure is set up in the model gives an explanation for this phenomenon. As the fixed cost of transferring funds stays unchanged while future cash flows decrease, the effect of the fixed cost (in relative terms) gets larger. Therefore, it is more efficient to hold on to the cash. The rise of both the upper trigger and the optimal target indicate exactly this. The target gets further away from the lower trigger, and the upper trigger gets further away from target. Therefore, the probability for the need of transferring funds decreases. Another interpretation derives form the definition of the beta. The beta can be understood as measure of the risk premium of the company. Thus, increasing risk requires larger cash buffer to set off the increasing possibility of liquidity shortage. The exponential shape of the change of the target level remains a puzzle.

While the discount factor β is eventually set by the shareholders of the company, the volatility of the cash flow is an exogenous parameter that can be controlled only to a certain extent. Hence, the next important step is to analyse the sensitivity of the model with regard to the volatility of the underlying cash flow. The sensitivity analysis of the volatility proceeds in the same manner as the analysis of the discount factor. First, the values of the objective function (*y-axes*) are plotted against the varying values of volatility (*x-axes*). *Figure 14* below indicates that the objective function increases with regard to volatility. This result is fairly clear. When volatility increases, the number of cash adjustments increase. As the cash adjustments come with a price tag, the discounted expected cost should rise with volatility.

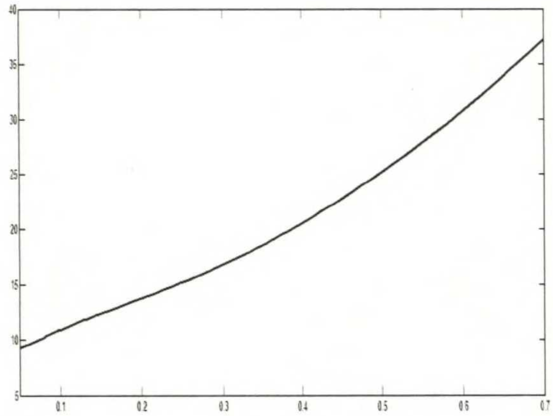


Figure 14: Objective function sensitivity with regard to cash flow volatility

Next, the optimal ICB policy is calculated for different volatilities. The optimal ICB policy corresponding to each volatility is reported in *Table 10*.

Table 10: Sensitivity of the optimal ICB policy with regard to cash flow volatility

σ	s^*	S_u	$R(\cdot)$
0.30	0.4034	1.2665	16.7461
0.32	0.4195	1.3186	17.4896
0.34	0.4351	1.3698	18.2205
0.36	0.4503	1.4200	18.9396
0.38	0.4653	1.4693	19.6477
0.40	0.4799	1.5178	20.3455

As before, the corresponding changes in the optimal IBC policy with regard to changing volatility are plotted in a graph. As could be expected, the optimal policy parameters grow as volatility grows. Increasing volatility increases the probability of crossing the upper or lower trigger level. As transferring the money has a cost, the model optimizes the total discounted cost by pulling the triggers further away from the target, hence lowering the probability of crossing the trigger level. Both the trigger and target values develop now in a linear manner.

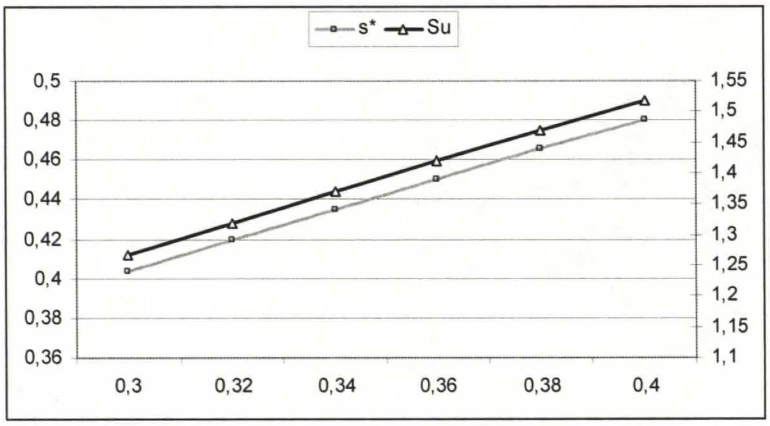


Figure 15: Optimal trigger and target levels with regard to cash flow volatility

An interesting conclusion could be made based on the sensitivity analysis above. If the volatility of the cash flow increases and the company is reluctant to adjust the optimal ICB policy to adapt to the change, the ICB policy could be kept unchanged by altering the discount factor β. The company cannot alter the cost of the debt that it is bearing; thus, the only way to alter β is by altering the cost of equity. By definition, the cost of equity is the expected return of equity set by the shareholders. Therefore, the only way to alter β is either by a change of shareholder sentiment or, more likely, by a change of the share price to a level that reflects the new value of the beta. Respectively, if the volatility of cash flow goes down and the ICB policy is kept unchanged, the company should become a more interesting investment opportunity. To conclude, one can state that if the company is reluctant to adjust their ICB policy as the volatility of the cash flow alters, the value of company's share should bear the burden. On the other hand, if the company alters their ICB policy in accordance to the volatility, the share price should remain unchanged.

As the final step of the sensitivity analysis, the effect of the proportional cost of upward and downward adjustments is analyzed. Different values of proportional costs are crosstabulated in *Table 11* below. For each pair of costs, the optimal target level, upper trigger level and corresponding objective function value are estimated. Noteworthily, the ICB algorithm is unable to calculate one cost pair (1.3;1.4). The reason behind this error remains a puzzle.

Table 11: Optimal ICB policy with regard to proportional adjustment costs

k_l proportional cost of lower adjustment	k_u proportional cost of upper adjustment: 1	1.1	**1.2**	1.3	1.4	1.5
1	0.4361 1.3626 18.0245	0.4352 1.3637 18.0460	0.4343 1.3648 18.0673	0.4333 1.3659 18.0887	0.4324 1.3671 18.1099	0.4324 1.3671 18.1099
1.1	0.4356 1.3619 18.0560	0.4346 1.3630 18.0775	0.4337 1.3641 18.0988	0.4328 1.3653 18.1210	0.4318 1.3664 18.1414	0.4309 1.3676 18.1626
1.2	0.4350 1.3612 18.0875	0.4341 1.3623 18.1089	**0.4331** **1.3635** **18.1303**	0.4322 1.3646 18.1516	0.4312 1.3658 18.1728	0.4303 1.3669 18.1940
1.3	0.4345 1.3605 18.1190	0.4335 1.3617 18.1404	0.4326 1.3628 18.1617	0.4316 1.3640 18.1830	0.4307 1.3651 18.2042	0.4297 1.3663 18.2254
1.4	0.4339 1.3599 18.1505	0.4329 1.3610 18.1718	0.4320 1.3622 18.1932	N/A	0.4301 1.3645 18.2356	0.4292 1.3657 18.2567
1.5	0.4333 1.3592 18.1819	0.4324 1.3604 18.2033	0.4314 1.3615 18.2246	0.4305 1.3627 18.2458	0.4295 1.3638 18.2670	0.4286 1.3650 18.2881

The table reveals three important results. First, the proportional cost does not appear to have a significant effect on the optimal policies. As the cost changes, the corresponding changes in the optimal policy are only marginal. Intuitively, this means that the holding cost and fixed transaction costs have such a significant contribution to the optimum that changes in the proportional cost become negligible. The small changes in the objective function support this argument. Second, as the value of the upper adjustment cost increases, i.e. the proportional price of investing excess cash increases, the upper trigger increases and the target decreases, thus widening the upper range of the allowed cash level. It should be noticed that the lower range narrows simultaneously. Finally, when the lower adjustment cost increases - i.e. the proportional price of attaining additional funds goes up - the upper trigger value and the target level both decrease, but asymmetrically. The upper range of the allowed cash level widens and the lower range narrows. The intuition behind this outcome is not as straightforward as one would desire. The argument stated here is that the relative size of the fixed cost gets smaller as the proportional cost gets larger. As the fixed element has a significant effect for the optimal, it is the indirect effect to the fixed element that causes the effect. The narrowing lower range compared to the widening upper range could be explained by higher probability of the downward jumps. Larger amount of the downward jumps reflects as lager indirect effect of the fixed component. To test the reasoning, one can look at a model where the lower fixed cost K_u *ceteris paribus* decreases.

Table 12: Optimal ICB policy with regard to lower fixed cost

	$K_u = 1$	$K_u=0,9$	$K_u=0,8$
s^*	0.4331	0.4122	0.3900
S_u	1.3635	1.3432	1.3215
$R(\cdot)$	18.1303	17.7028	17.244

The results reported in *Table 12* support the statement above and confirm the argument about the significant contribution of the fixed cost element for the entire model. To conclude, one can state that for the model, the changes of the proportional cost do not have a significant effect to the optimal ICB policy. On the other hand, the fixed cost element seems to have a much larger effect, both direct and indirect. This naturally raises further questions about the correct level of the fixed cost. Solving these questions is at this point left for further studies.

This concludes the analysis of the optimal ICB policy. Some criticism towards the model is in place before advancing to the final chapter that summarises the results of this work. The model used in the analysis above is general in nature and can be adjusted to accommodate for different types of jump size distributions (Bar-Ilan et al. 2004). Despite the general form, the model is not stable in empirical surroundings. Looking at the matrix (33), one can easily see that large sizes of S are out of the feasible range when solving the optimum. The model also ignores a number of costs that may be incorporated to additional financing. Kim et al. (1998) name such as legal fees, accounting and printing cost and underwriter fees. They also argue that these cost components may have significant economics of scale. The model also neglects the lead times of external financing. Lead times have a significant effect if loan instalments can be made only at predetermined times. The arguments originally set by Baumol (1952) of dealing with only a single economic unit thus neglecting interactions between various demands of cash in the economy and ignoring precautionary and speculative demands of cash are also relevant for this work. To conclude, one can say that despite the generalized form of the model, it still lacks a number of realistic variables and therefore the model is, and should be regarded as a significantly simplified view of reality. Bealey and Myers (2003) argue that no model will ever succeed in providing a substitute for the judgement of a cash manager. However, the model gives a good picture of the general levels of an optimal ICB policy, and even though the results are what one may call ballpark figures, they are still much more indicative than no result at all.

VI Summary and Conclusions

In this empirical study, the solution for the main research question of finding an optimal cash management policy for a corporation was found by solving three predetermined sub-objectives of (i) choosing the relevant cash management model, (ii) estimating or by other means defining the relevant parameters for the chosen model and (iii) solving and analysing the model.

First, it is argued that even though the precautionary motive of cash management has gained ground in recent research (Opler et al. 1999), there is still room for transaction-cost motive models, especially when the optimization problem is set to a short run-time interval. It is also argued that these two motives originally introduced by Keynes (1936) could in fact be seen as long-run and short-run approaches to the same cash management problem. The precautionary motive builds on managing liquidity by paying dividends and issuing new equity (Anderson and Carverhill 2005). These can be defined as long-term strategic decisions; thus, not feasible for daily cash management operations. Therefore, the trigger-target-models provide a superior basis for liquidity management in the short run. The study of both motives is argued to be relevant for cash management purposes.

The original trigger-target model was introduced by Baumol (1952) and Miller and Orr (1966). Recently various authors (Hindler and Waldman 2001; Green 2001; Premachandra 2004; Bar-Ilan et al. 2004) have studied different expansions to the original models. Harrison et al. (1983) show that for the ICB type cash management model, an optimal policy exists when the underlying cash flow is assumed to follow the Brownian Motion. Bar-Ilan et al. (2004) generalize the model to accommodate jump diffusion as an underlying stochastic process. In this study, the general model of Bar-Ilan et al. (2004) is used to solve the optimal ICB policy. In order to do so, the parameters of the underlying stochastic process are estimated.

The sample data, measured in log returns, witness skewness and leptokurtic behaviour, therefore, jump-diffusion model can be used to describe the time series (Merton 1976; Kou 2002; Chacko and Viveira 2003; Hanson and Zhu 2004; Ramedani and Yeng 2006). Several arguments are made to conclude that for the sample data, the Double Exponential Jump Diffusion (DEJD) is an adequate choice to model the distribution of the jump size. The sample data is also argued to give a good presentation of the cash flow risk faced by the company.

With stochastic process parameters, the optimal ICB policy is solved using the *Generalized Impulse Control Model of Cash Management* by Bar-Ilan et al. (2004). Although the model is argued to be only a simplification of the real world, the results are argued to give good guidelines for cash management. The optimization result indicates that at the optimum, with the given cash flow process supporting earlier findings of Hanson et al. (2004), the upper and lower target levels are equal and 1.43 times the size of the lower trigger set by the company. The corresponding upper trigger is 2.36 times the size of the lower trigger.

Extensive sensitivity analysis reveals that proportional transfer costs have a minor effect on the optimal policy. On the other hand, fixed transfer costs have a significant effect, both direct and indirect, on the optimal policy. The upper target level is found to be an increasing function of volatility. This result is consistent with the results of Fenkel and Jovanovic (1981) and Bar-Ilan et al. (2004). In addition, the optimal target level is also found to be an increasing function of volatility. The optimal target level and upper trigger level are also found to be increasing functions of the discount factor beta.

The findings of this thesis give rise to a number of recommendations for further studies. First, the argument that the precautionary motive and the transaction cost motive could in fact be seen as a long-run and short-run optimization problem of cash management would require some additional research. Frenkel and Jovanovic (1980) derive closed-form and steady-state solutions for optimal money holding in order to incorporate transaction cost and precautionary motives. However, as far as the author of this study is aware, there has not been an attempt to solve the trigger-target model of the transaction cost motive as a variation around the long-run optimal level set by a precautionary motive model. Second, further work is required to confirm that the chosen DEJD representation of the underlying stochastic process does in fact return the best possible fit for the time series. Other possible time series representations, such as ARGH and GARGH models where volatility is not constant, should also be considered. Further, the results in this thesis state that the optimal policy is an increasing function of cash flow volatility. Therefore, the causes of the volatility, e.g. correlation between raw material price volatility and cash flow volatility, should be studied. Additionally, the correlation between forecasted and realized cash flows should be studied for better description of the cash flow uncertainty. Finally, as this study provides a static solution for the optimal cash management problem, the parameters of the underlying process should be regenerated from time to time.

Appendix

A1 Itô's Lemma

Suppose that x follows the generalized Wiener process in which the parameters a and b are functions of the underlying variable x and time t. This type of stochastic process is known as an *Itô process*

(a 1) $$dx=a(x,t)dt + b(x, t)dz$$

where dz is a Wiener process. The variable x has a drift rate of a and a variance rate of b^2. Itô's lemma[17] shows that a function of $G(x, t)$ follows the process

(a 2) $$dG=\left(\frac{\partial G}{\partial x}a+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2\right)dt+\frac{\partial G}{\partial x}bdz$$

Thus G is also an Itô process, with a drift rate of

(a 3) $$\frac{\partial G}{\partial x}a+\frac{\partial G}{\partial t}+\frac{1}{2}\frac{\partial^2 G}{\partial x^2}b^2$$

and a variance rate of

(a 4) $$\left(\frac{\partial G}{\partial x}\right)^2 b^2$$

[17] For complete proof, see Itô (1951)

A2 Lognormal property

If S follows a stochastic process

(a 5) $$dS = \mu S dt + \sigma S dz$$

with the constants μ and σ, and $ln\ S$ is defined as

(a 6) $$G = ln\ S.$$

Since

(a 7) $$\frac{\partial G}{\partial S} = \frac{1}{S}, \quad \frac{\partial^2 G}{\partial S^2} = \frac{1}{S^2}, \quad \frac{\partial G}{\partial t} = 0.$$

Using Itô's lemma (Appendix A1)

(a 8) $$dG = \left(\mu - \frac{\sigma^2}{2} \right) dt + \sigma dz$$

With the constants m and s, the equation (a 8) above indicates that G = ln S follows a generalized Wiener process. The change in $ln\ S$ between 0 and some future point of time T is normally distributed with a mean of $(\mu - \sigma^2/2)T$ and variance of $\sigma^2 T$. (Hull 2006, 274-275.)

A3 Second order approximation of bin probability distribution

Hanson et al. (2004) introduced the Multinomial Maximum likelihood estimation for binned data. The estimation method leans on the second order approximation of the bin probability [x1,x2] that is given by

(a 9) $$\Phi_{dejd}(x_1,x_2) \cong \frac{\sum_{k=0}^{2} p_k(\lambda\Delta t)\Phi_{dedj}^{(k)}(x_1,x_2)}{\sum_{j=0}^{2} p_j(\lambda\Delta t)}$$

for $-\infty < x < \infty$.

(a 10) $$\Phi_{dejd}^{(0)}(x_1,x_2) \equiv \Phi_n(x_1,x_2;\mu,\sigma^2)$$

where $\Phi_n(x_1,x_2;\mu,\sigma^2)$ is the normal distribution of $[x_1, x_2]$, $\mu \equiv \mu_{ld}\Delta t$, $\sigma \equiv \sqrt{\sigma_d^2 \Delta t}$, $p_k(\Lambda) = e^{-\Lambda}\Lambda^k / k!$ is the Poisson distribution with the parameter $\Lambda = \lambda\Delta t$, k jumps and corresponding time increment in years Δt.

(a 11) $$\Phi_{dejd}^{(1)}(x_1,x_2) = \Phi_n(x_1,x_2;\mu,\sigma^2) + p_1\left(\rho_{x_2,v_1} - \rho_{x_1,v_1}\right) + p_2\left(\rho_{x_1,v_2} - \rho_{x_2,v_2}\right),$$

where

$$\begin{aligned}
v_1 &= \mu - 0.5\sigma^2 / \mu_1, \\
v_2 &= \mu + 0.5\sigma^2 / \mu_2, \\
\rho_{x_2,v_1} &= e^{+(x_2-v_1)/\mu_1}\Phi_n\left(-x_2;-\mu+\sigma^2/\mu_1,\sigma^2\right), \\
\rho_{x_1,v_1} &= e^{+(x_1-v_1)/\mu_1}\Phi_n\left(-x_1;-\mu+\sigma^2/\mu_1,\sigma^2\right), \\
\rho_{x_1,v_2} &= e^{-(x_1-v_2)/\mu_2}\Phi_n\left(x_1;\mu+\sigma^2/\mu_2,\sigma^2\right), \\
\rho_{x_2,v_2} &= e^{-(x_2-v_2)/\mu_2}\Phi_n\left(x_2;\mu+\sigma^2/\mu_2,\sigma^2\right),
\end{aligned}$$

(a 12)

$$
\begin{aligned}
&\Phi^{(2)}_{dejd}(x_1,x_2)=\Phi_n(x_1,x_2;\mu,\sigma^2)\\
&+\mu_1\left(\left(p_{12}+p_{11}\left(\mu-\frac{\sigma^2}{\mu_1}+\mu_1-x_2\right)\right)\rho_{x_2,v_1}-\left(p_{12}+p_{11}\left(\mu-\frac{\sigma^2}{\mu_1}+\mu_1-x_1\right)\right)\rho_{x_1,v_1}\right)\\
&+\mu_2\left(\left(p_{12}-p_{22}\left(\mu+\frac{\sigma^2}{\mu_2}-\mu_2-x_1\right)\right)\rho_{x_1,v_2}-\left(p_{12}-p_{22}\left(\mu+\frac{\sigma^2}{\mu_2}-\mu_2-x_2\right)\right)\rho_{x_2,v_2}\right)\\
&+\frac{\sigma}{\sqrt{2\pi}}(\mu_2 p_{22}-\mu_1 p_{11})\left(e^{-z_1^2/2}-e^{-z_2^2/2}\right),
\end{aligned}
$$

where

$p_{11}=(p_1/\mu_1)^2,\ p_{22}=(p_2/\mu_2)^2,\ p_{12}=2p_1p_2/(\mu_1+\mu_2),\ z_1=(x_1-\mu)/\sigma,\ z_2=(x_2-\mu)/\sigma.$

For DEJD, the density of the jump-amplitude is

(a 13) $$\phi_Q(q)=\frac{p_1}{\mu_1}\exp\left(\frac{q}{\mu_1}\right)I_{\{q<0\}}+\frac{p_2}{\mu_2}\exp\left(\frac{-q}{\mu_2}\right)I_{\{q\geq 0\}}$$

where μ_1 and μ_2 are one-sided means and p_1 is the probability of downward jumps and $p_2=1-p_1$ the corresponding probability of upward jumps. The set indicator function is $I_{\{S\}}$ for set S. Q has moments,

$$
\begin{aligned}
\mu_j &= E_Q[Q]=-p_1\mu_1+p_2\mu_2\\
\sigma_j^2 &= Var_Q[Q]=p_1\left((\mu_j+\mu_1)^2+\mu_1^2\right)+p_2\left((\mu_j+\mu_2)^2+\mu_2^2\right)
\end{aligned}
$$

A4 The MATLAB code for GBM estimation

```
function G = myfunct(q)
a = [-0.36:0.01:0.34]';          %theoretical bin range
b = [-0.35:0.01:0.35]';          %theoretical bin range
n=563;                           %number of observations

X = normcdf(b,q(1),q(2));        %normal distributions with estimated
                                 %parameters
Y = normcdf(a,q(1),q(2));        %normal distributions with estimated
                                 %parameters

Z=max(X-Y,eps);                  %positivity constraint
H=n*Z;                           %number of observations per
                                 %theoretical bin
G=-sum(obs.*log(H));             %objective function (obs refers to the
                                 %observed data)
```

A5 The MATLAB code for DEJD estimation

```
function D = myfunct(q)
x1 = [-0.36:0.01:0.34]';         %theoretical bin range
x2 = [-0.35:0.01:0.35]';         %theoretical bin range
n=563;                           %number of observations

l = q(1);                        %parameter estimate for the number of
                                 %jumps
p1 = q(2);                       %parameter estimate for the
                                 probability of %jumps down
m1 = q(3);                       %parameter estimate for the mean of
                                 %downward jumps
m2 = q(4);                       %parameter estimate for the mean of
                                 %upward jumps
t = 0.003558;                    %time increment in years
M1 = 0.004673;                   %mean of the sample data
M2 = 0.003044;                   %variance of the sample data

%following equations calculate rest of the model parameters

p2 = 1-p1;                       %probability of upward jumps
L=l*t;                           %intensity rate of the CPP
P0=exp(-L)*L.^0/factorial(0);
P1=exp(-L)*L.^1/factorial(1);
P2=exp(-L)*L.^2/factorial(2);

mj = -p1*m1 + p2*m2;
s2j = p1*((mj + m1).^2 + m1.^2) + p2*((mj + m2).^2 + m2.^2);
m1d = (M1 - mj*L)/t;
s2d =max((M2-(s2j+mj.^2)*L)/t,eps);
m=m1d*t;
s=sqrt(s2d*t);
```

```
v1=m-0.5*s.^2/m1;
v2=m+0.5*s.^2/m2;

r21 = exp((x2-v1)/m1).*normcdf(-x2,(-m+s.^2/m1),s);
r11 = exp((x1-v1)/m1).*normcdf(-x1,(-m+s.^2/m1),s);
r12 = exp(-(x1-v2)/m2).*normcdf(x1,(m+s.^2/m2),s);
r22 = exp(-(x2-v2)/m2).*normcdf(x2,(m+s.^2/m2),s);

p11 = (p1/m1).^2;
p22 = (p2/m2).^2;
p12 = 2*p1*p2/(m1+m2);

z1 = (x1-m)/s;
z2 = (x2-m)/s;

y1 = m-s.^2/m1+m1-x2;
y2 = m-s.^2/m1+m1-x1;
y3 = m+s.^2/m2-m2-x1;
y4 = m+s.^2/m2-m2-x2;

X1 = normcdf(x1,m,s);
X2 = normcdf(x2,m,s);

F0=X2-X1;
F1=F0+p1*(r21-r11)+p2*(r12-r22);
F2=F0+m1.*((p12+p11.*y1).*r21-(p12+p11.*y2).*r11)+m2.*((p12-
p22.*y3).*r12-(p12-p22.*y4).*r22)+s/sqrt(2*pi)*(m2.*p22-
m1.*p11)*(exp(-z1.^2/2)-exp(-z2.^2/2));

F=(P0*F0+P1*F1+P2*F2)/(P0+P1+P2);

H=n*F;                               %number of observations in a
                                     %theoretical bin
D=-sum(obs.*log(H));                 %objective function where obs %refers
                                     to observations of the %sample data
```

A6 The MATLAB code for the Generalized Impulse Control Model

```
function G = myfunct(w)
sl=w(1);                          %parameter estimate for the lower
                                  %target level
su=w(2);                          %parameter estimate for the upper
                                  %target level
x=sl;                             %starting cash level at time t=0
S=w(3);                           %parameter estimate for the upper
                                  %trigger level

b=0.05; b0=0;                     %value of beta

                                  %parameters of the stochastic process
s=0.33751392;                     %volatility
m=0;                              %drift rate
l=0.514687376;                    %intensity of downward jumps
v=30.03003003;                    %mean of downward jumps
n=0.558472291;                    %intensity of upward jumps
e=24.50980392;                    %mean of upward jumps

h0 = 1;                           %holding cost of liquid assets
ku = 1.2;                         %proportional transfer cost up
kl = 1.2;                         %proportional transfer cost down
Ku = 1;                           %fixed transfer cost up
Kl = 1;                           %fixed transfer cost down


%calculates the roots for the function (xx)

syms a;
fa = ((s^2*a^2/2)*(v+a)*(e-a)) - (m*a*(v+a)*(e-a))-
((l*a/(v+a))*(v+a)*(e-a)) + ((n*a/(e-a))*(v+a)*(e-a))- b*(v+a)*(e-a);
fa1=collect(fa,a);
fa2=sym2poly(fa1);
ab=sort(roots(fa2), 'descend');


%calculates the roots of the function (xx)when b=0

syms a;
fa0 = ((s^2*a^2/2)*(v+a)*(e-a)) - (m*a*(v+a)*(e-a))-
((l*a/(v+a))*(v+a)*(e-a)) + ((n*a/(e-a))*(v+a)*(e-a))- b0*(v+a)*(e-a);
fa01=collect(fa0,a);
fa02=sym2poly(fa01);
ab0=sort(roots(fa02), 'descend');

%forms the matrix A(b) (xx)

AB =[1 e/(e-ab(1))  exp(-ab(1)*S)  v/(v+ab(1))*exp(-ab(1)*S)
    ; 1 e/(e-ab(2))  exp(-ab(2)*S)  v/(v+ab(2))*exp(-ab(2)*S)
    ; 1 e/(e-ab(3))  exp(-ab(3)*S)  v/(v+ab(3))*exp(-ab(3)*S)
    ; 1 e/(e-ab(4))  exp(-ab(4)*S)  v/(v+ab(4))*exp(-ab(4)*S)];
```

```
%forms the matrix A(0)(xx)

AB0 =[1 e/(e-ab0(1))  exp(-ab0(1)*S)  v/(v+ab0(1))*exp(-ab0(1)*S)
    ; 1 e/(e-ab0(2))  exp(-ab0(2)*S)  v/(v+ab0(2))*exp(-ab0(2)*S)
    ; 1 e/(e-ab0(3))  exp(-ab0(3)*S)  v/(v+ab0(3))*exp(-ab0(3)*S)
    ; 1 e/(e-ab0(4))  exp(-ab0(4)*S)  v/(v+ab0(4))*exp(-ab0(4)*S)];

%solves the additive components Fx(b)for different values of x and b

FxB = inv(AB)*[exp(-ab(1)*x)  exp(-ab(2)*x)  exp(-ab(3)*x)  exp(-
ab(4)*x)]';
Fx0 = inv(AB0)*[exp(-ab0(1)*x)  exp(-ab0(2)*x)  exp(-ab0(3)*x)  exp(-
ab0(4)*x)]';

FslB = inv(AB)*[exp(-ab(1)*sl)  exp(-ab(2)*sl)  exp(-ab(3)*sl)  exp(-
ab(4)*sl)]';
Fsl0 = inv(AB0)*[exp(-ab0(1)*sl)  exp(-ab0(2)*sl)  exp(-ab0(3)*sl)
exp(-ab0(4)*sl)]';

FsuB = inv(AB)*[exp(-ab(1)*su)  exp(-ab(2)*su)  exp(-ab(3)*su)  exp(-
ab(4)*su)]';
Fsu0 = inv(AB0)*[exp(-ab0(1)*su)  exp(-ab0(2)*su)  exp(-ab0(3)*su)
exp(-ab0(4)*su)]';

%counts the value of function f(a)  (xx)

Fa = s^2*a^2/2 - m*a - l*a/(v+a) + n*a/(e-a);

%defines the expected values Ex and Esu

Ex = (-exp(-a*x)+FxB(1)+e/(e-a)*FxB(2)+exp(-a*S)*FxB(3)+exp(-
a*S)*v/(v+a)*FxB(4))/(Fa-b);
Esu = (-exp(-a*su)+FsuB(1)+e/(e-a)*FsuB(2)+exp(-a*S)*FsuB(3)+exp(-
a*S)*v/(v+a)*FsuB(4))/(Fa-b);

%defines the cost function ex(a,b)(xx)and differentiates the cost
function

exAB = Ex+((Fx0(3)+Fx0(4))/(Fsu0(1)+Fsu0(2)))*Esu;
DexAB = diff(exAB);

%defines the holding cost functional  (xx)

HxB = -subs(DexAB, 0);

%defines the discount factor functionals TxB and FFxB

TxB = 1-b*subs(exAB, 0);
FFxB= FxB(3)+FxB(4)+((FslB(3)+FslB(4))*(FxB(1)+FxB(2)))/(1-FslB(1)-
FslB(2));
TslB = FslB(1)+FslB(2)+((FslB(1)+FslB(2))*(FslB(3)+FslB(4)))/(1-
FslB(3)-FslB(4));
TsuB = FsuB(1)+FsuB(2)+((FsuB(1)+FsuB(2))*(FsuB(3)+FsuB(4)))/(1-
FsuB(3)-FsuB(4));
```

```
FFslB= FslB(3)+FslB(4)+((FslB(3)+FslB(4))*(FslB(1)+FslB(2)))/(1-
FslB(1)-FslB(2));
FFsuB= FsuB(3)+FsuB(4)+((FsuB(3)+FsuB(4))*(FsuB(1)+FsuB(2)))/(1-
FsuB(1)-FsuB(2));

%defines the discounted holding cost function (xx)

E1 = HxB/(1-TxB);

%defines the proportional and fixed cost functions (xx-xx)

E2 = (FFxB*(Fx0(3)*(S-su)+Fx0(4)*(S-su+1/v)))/(1-FFsuB*(Fsu0(3)*(S-
su)+Fsu0(4)*(S-su+1/v)));
E3 = (TxB*(Fx0(1)*sl+Fx0(2)*(sl+1/e)))/(1-
TslB*(Fsl0(1)*sl+Fsl0(2)*(sl+1/e)));
E4 = FFslB/max((1-FFsuB),eps);
E5 = TslB/(1-TslB);

%the objective function i.e. discounted costs

R = h0*E1+ku*E2+kl*E3+Ku*E4+Kl*E5;
```

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